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Newsletter

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IMTech Newsletter

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One-hundred years of spin

Contents

Editorial

In this issue we interview Sònia Fernández on the occasion of her recent appointment as Full Professor [▷]; Josep Díaz, Emeritus Full Professor [▷]; Jordi Guàrdia [▷], after having been awarded the 25th UPC Prize for Teaching Initiatives and the Vicenç Vives Award for Teaching Quality, and Josep M. Rossell & Núria Salan [▷], after having been distinguished *ex-aequo* for their teaching careers with the UPC Prize for Quality in University Teaching —J. M. Rossell has also received the Vicenç Vives Award for Teaching Quality; and Isiah Zaplana [▷], on the occasion of taking office with a María Zambrano Research Fellowship.

Next we have two Research focus notes, three PhD highlights and one Outreach contribution. Victor Rotger presents *Darmon's conjecture on Stark-Heegner points* [▷] and Jordi Tura introduces his work on *Bell correlations in quantum many-body systems* [▷]. Then follow the thesis summaries of Mar Giralt [▷], on *Homoclinic and chaotic phenomena to L_3 in the Restricted 3-Body Problem*; of Tuomas Hakoniemi [▷], on *Size bounds for algebraic and semialgebraic proof systems*; and of Guillem Belda [▷], on *Conformal Marked Bisection for Local Refinement of n-Dimensional Unstructured Simplicial Meshes*. The Outreach contribution is by Pablo Sáez [▷] with the title *Why and how do cells migrate?*.

In the Chronicles section, the first five are devoted to the ICM-2022: four to the Fields Medals lectures and one to the Chern Medal. The contributions about the Fields Medals are by Juanjo Rué [▷], on James Maynard; by Joaquim Ortega Cerdà [▷], on Maryna Viazovska; by Guillem Perarnau [▷], on Hugo Duminil-Copin; and by Anna de Mier & Sebastià Xambó [▷], on June Huh. The contribution on the Chern Medal, awarded to Barry Mazur, is by Joan C. Lario [▷].

There are four additional Chronicles: two by Gemma Huguet, on the **IMTech** Fall Colloquium [▷] and on four **IMTech** scholarships [▷]; one by Marc Noy and Lluís Vena [▷], on the conference RandNET; and one by Antonio Huerta [▷] on the Honorary Doctorate to Jaume Peraire.

The graduation ceremonies 2021-2022 of the FME [▷] and CFIS [▷], and the renewed Board of Directors of the SCM [▷] are reported in the Events section.

Two mathematical breakthroughs achieved in 2022 are discussed in the Reviews section. The first review, by S. Xambó, focuses on the the paper *Interpolation of Brill–Noether curves*, by Eric Larson and Isabel Vogt [▷]. The second, by Jordi Castellví and Miquel Ortega, reviews the paper A Proof of the Kahn-Kalai Conjecture, by Jinyoung Park and Huy T. Pham [▷].

In the Editorial of the NL03, we celebrated several important recent distinctions of Eva Miranda, including the Bessel Prize 2022 awarded by the Alexander von Humboldt Foundation. To that record we are glad to add the François Deruyts Prize of the Académie Royale de Belgique, "for her important contributions to the study of fully integrable systems and to the development of theory of mechanical systems admitting singularities". This prize is awarded every four years since 1902 and Miranda is the second woman (the first was Simone Gutt, 1998) enriching the list of prestigious mathematicians of previous editions, among them Jacques Tits (1962) and Pierre Deligne (1974). The picture below features the reception of the Bessel and the Deruyts prizes.

The *laudatio* in the award ceremony of the Bessel Prize stated that "Professor Miranda is an outstanding mathematician who has made significant contributions to Poisson and singular symplectic geometry as well as geometric quantisation. She has received wide international recognition for her groundbreaking work in fluid dynamics, concerning Turing complete Euler flows and the Navier–Stokes equations. During her stay in Germany, Professor Miranda's research will focus on questions at the interface of symplectic topology and dynamical systems". Here it is fitting to recall the Research focus note contributed by R. Cardona, E. Miranda and D. Peralta-Salas in the NL01 (page 9) on Undecidable fluid particle paths and 3D fluid computers.

Jezabel Curbelo (DMAT and **IMTech**) has obtained one of the Leonardo Scholarships awarded by the BBVA Foundation in the field of Mathematics. The Leonardo Scholarships are intended to support personal projects of researchers and cultural creators in intermediate stages of their career, between 30 and 45 years old, who are characterized by significant scientific production. J. Curbelo was interviewed in the NL01 (page 3). Our congratulations for this new recognition.

We wish all readers a most Happy 2023!

Interviews

Sonia Fernández-Méndez has recently won a position of Full Professor in Applied Mathematics at the Universitat Politècnica de Catalunya (UPC). She is member of **IMTech**, of the research group LaCàN, and of the Department of Civil and Environmental Engineering (DECA).

She pursued a "Llicenciatura en Matemàtiques" at UPC in 1992 (the first year that this program was offered) and obtained her degree in four years, in 1996. After that, she got a position as full-time lecturer in the department of Applied Mathematics III at UPC (now included in DECA), and started her PhD in the Applied Mathematics program under the supervision of Antonio Huerta, defending her thesis entitled *Mesh-free methods and finite elements: friend or foe?* in 2001. As a result of her thesis and posterior early research, she was awarded with the UPC Outstanding Thesis Award 2001; the Jacques Louis Lions Award 2010 for Young Scientists in the field of Computational Mathematics, by the European Community on Computational Methods in Applied Sciences (ECCOMAS); and the Carles Simó award 2009 for young scientist in the field of Computational Mechanics, by the Sociedad Española de Métodos Numéricos en Ingeniería (SEMNI).

Her research activity has always been mainly developed in the field of Computational Modeling with Partial Differential Equations (PDEs) and, in particular, in the derivation, study and application of novel advanced discretization methods. She has supervised seven PhD theses and participated in about thirty research projects, three of them funded by the European Community.

She served as Academic Secretary and as Coordinator of the Master and PhD programs in Mathematics in the Dean team of the Facultat de Matemàtiques i Estadística (FME) for about ten years, and currently she is the Academic Secretary of the DECA.

NL. ***Our congratulations for your recent appointment to Full Professor in Applied Mathematics. Having started your university studies in 1992 as a member of the first recruitment for the five-year degree in Mathematics offered by the FME of UPC, we would like to begin by asking your recollections about when and how did you decide that choice.***

The teacher for mathematics in my last year in high-school (COU, "university orientation course") was crucial in the decision. She did some simple proofs during her lectures and I was fascinated by the conciseness and clarity of a mathematical proof. She was also the person that commented to me about the new degree that was offered at UPC, and that it was supposed to have a more applied character than other degrees in mathematics. The fact that the FME was closer to my parents' home than other faculties was also helping in the decision. I was also very interested in Architecture, but I am really glad that I finally chose mathematics, since I enjoyed it very much.

For the first cohort, that degree in Mathematics was a rather singular experience, for students and faculty alike. What are your most vivid memories of those years? In particular, how did you manage to complete the degree in four years?

I have very good memories of those four years from both academic and social points of view. I met my husband and very good friends at FME. I had very much fun at the "Delegació" (the student meeting facilities) and at the computer rooms. Since we were few students, during the first academic years, we could play network computer games like Doom there. I also heard about internet for the first time seeing how one student was connecting to a server from a computer in room PC2. He explained to me what he was doing and I did not understand it until a couple of years later. From the academic point of view, the first lecture was Calculus and it was quite a shock to see that we were supposed to proof things that seemed to be so obvious. The first weeks I had the feeling that I was going to fail every topic, but in the end, once I adapted, it became natural. Lecturers were amazing. You could see that they were also excited and very glad to teach the degree in Mathematics. The only minor disadvantage was that the exams where a kind of lottery: some were really difficult and long (5 hours in some cases) and some were too easy, because lecturers did not have a previous reference. We also did not have exams from previous years, thus there was no way to predict what kind of exam we were going to see. About completing the degree in four years, I was really not aware of it. I thought the degree was a standard four-years degree. It was afterwards that I new that the credits corresponded to five years and they were condensed.

When did you discover that you would be a researcher? At that moment, what fields were in your horizon?

I think that my thesis supervisor, Prof. Antonio Huerta, knew before myself that I was going to be a researcher. Initially, his department hired me as full-time associated professor and, at some point, I started a PhD thesis under his supervision. I was really lucky to have him show me how intense and fascinating research can be.

What problems did you aim at for your thesis research?

It dealt with a novel family of numerical methods for PDEs known as *mesh-less*, *mesh-free* or *particle methods*, that became very popular in the early 2000s. It was a very interesting thesis topic but few years after the thesis defense I moved to Discontinuous Galerkin (DG) Finite Element Methods. I have been working in advanced discretization techniques (DG, extended-FEM, etc) since then, recently with special interest in problems modeled with fourth-order PDEs and interface or crack problems.

Your thesis covers theoretical, computational and applied aspects. Can you describe your main contributions?

The results of the thesis had quite an important impact in the Computational Mechanics community at the moment of publication, leading to an invitation to be coauthor of the chapter on meshless methods of the Encyclopedia of Computational Mechanics. However, the publication that has finally lead to more citations, and I think has been useful for more researches, is a subproduct of the thesis where we explain and study different existing techniques for imposing Dirichlet boundary conditions in weak form.

What significance have had in your career the awards you have received?

The two awards are for young researchers; thus they are related to the research I did with Prof. Huerta. They gave me visibility in the Computational Mechanics community.

You also have a rich record in teaching. What is you present view of that endeavor?

Teaching is my most important contribution to the community. Even though I am proud of the citations of my papers in indexed journals, the impact of teaching on students is much more important to me. I enjoy teaching at bachelor and master levels, and also supervising PhD thesis. I would like to think that I contribute to prepare them to be excellent professionals and researchers.

As for university administration, you also have had your fair share and we would like to know your reflections on this aspect of academic life.

I have been for about fourteen years devoting some of my time and attention to management, first in the Dean team at FME and now as Secretary of the Department of Civil and Environmental Engineering. It has been very interesting to learn how administrative and academic procedures work from both points of view. Management is not the task I enjoy most, but collaborating with the management teams and administrative staff has always been a good personal experience. I would like to mention that, since the UTG's (Transversal Management Units) were created at UPC, it is especially important to work every day to maintain a good collaboration between administrative staff and lecturers. We have different and interesting ways to understand procedures, which in many cases are complementary and necessary.

Now that you have secured a Full Professorship at UPC, how do you envisage this new stage in your academic progression?

A former PhD student of mine told me that now I have the position corresponding to the tasks I have been doing in the last years. She may be right, because my activity, goals and commitment to all aspects of the academic life have not changed.

As a vintage student of the FME, with subsequent important teaching and academic responsibilities in it for many years, how do you see the future of that institution?

It is obvious that we have a problem with the grades required to access the bachelor in Mathematics. It does not make sense at all not being able to accept students with a grade under 13/14, especially when about 6 marks depend on topics not related in any way to Mathematics. I would prefer staying with a unique group of around 60 students per year, because lecturers get to know all of them and the FME can select lecturers to keep the excellent teaching quality, but we have a commitment with society and the need for mathematicians is continuously increasing.

As a member of IMTech, what strategies would you favor in order unfold its potential?

I have to admit that, given my involvement in university management in the last years, I have not paid enough attention to the activities to promote collaboration and get support for the Mathematics community at UPC. I am sincerely grateful to the people that are devoting a precious amount of time and attention to it. ▷Editorial

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Josep Díaz Cort is Emeritus Full Professor of Computer Science at the Computer Science Department at UPC where he has been Full Professor since 1984. He holds a BSc and MSc degrees in Mathematics from the UC Riverside (1975) and a PhD degree in Physics from the Universidad Literaria de Valencia (1981). He has been founder and the head of the Algorithmics, Bioinformatics, Complexity and Formal Methods of the UPC (ALBCOM).

His research focuses on Algorithmics, Graph Theory and Combinatorics, randomized processes and probabilistic techniques in Computer Science, and Computational Complexity. Through his academic trajectory he was the general or local UPC coordinator for many projects, in particular of ten European projects in the period (1992-2011).

He received the Spanish National Prize in Computer Science Jose Garcia Santesmases (2011) and the Senior Distinction for the promotion of the research given by the Catalan Government (2002). He is a Fellow of the European Association Theoretical Computer Science (2017) and member of the Academia Europaea (2011). He has served as Chair of the selection committee for EATCS-Fellows (2019) and the ACM Paris Kanellakis Award (2020), also as member of the selection committee for the Gödel Prize (ACM and EATCS), for outstanding papers in Theoretical Computer Science (2010-2013), and of the selection committee for EATCS-Fellows (2019-2024). He forms part of the "Comitè d'ètica de la recerca" in the Hospital Clinic de Barcelona since 2020.

He has been member of the ACM-Europe Council (2014-2017), one of the four external members in the Advisory Committee for the Japanese initiative New Horizons in Computing (2005-2008) and President of the European Association for Theoretical Computer Science (1997-1999-2002). Since 2012 he is in the Advisory Board of Springer book series Monographs in Theoretical Computer Science and Texts in Theoretical Computer Science. Has been founder and co-Editor-in-Chief of Computer Science Reviews and in the Editorial Board of Theoretical Computer Science (since 1995).

NL. ***Your background was originally in physics. How did you become interested in computer science?***

In the 70's, I was studying in the US. My basic graduate courses were Analysis, Algebra, Topology, Probability, although I also took several other different courses like Combinatorics, Introduction to Algorithms and data structures, an undergrad course in Physics, and a semester in Introduction to Quantum Mechanics, among others courses.

Towards the mid 70's, I was working towards the PhD in math, under Professor Larry Harper. He got an interesting isoperimetric graph problem related to minimizing the graph layout in the wiring of a big electronic circuit for a machine at the JPL. The first part of my thesis consisted in proving the problem was a NP-complete (possibly difficult to solve in polynomial time), and I also showed that for certain particular kinds of graphs, the problem was in P, by providing polynomial time algorithms.

The second question that Larry asked me was to use the model of Boolean circuits with gates $\{\vee, \wedge, \neg\}$, to prove that P=NP, by taking an NP-complete problem and showing that a polynomial size circuit could solve the problem. Obviously, I did not succeed in solving the P=NP? problem, but I got some nice exponential bounds for the Hamiltonian circuit on a graph, which were included in the manuscript.

After Karp's 1972 paper stating the first 21 NP-complete problems, the field of complexity of decidable problems became a very hot topic of research in many math departments, at least in the US universities. As it is well known, the P = NP? problem is still open, it boils down to deciding whether exhaustive search is necessary to solve problems. It is one of the seven millennium problems posed by the Clay institute. A gentle introduction to the problem is Chapter 3 of Keith Devlin's book: The Millennium Problems, Basic books, 2002. Another introduction is https://en.wikipedia.org/wiki/P_versus_NP_problem. There are plenty of other nice non technical surveys on that topic.

For personal and family reasons, I wanted to return quickly to Spain. Sometime around early 1979, I got an offer from Martí Vergés and Xavier Berenguer to come to the UPC. They asked me to start teaching the following September, in the incipient school of computer science (FIB). At the time it was impossible to validate a Doctorate done in the US into a Spanish doctorate, and at least from the US point of view it was considered very bad behavior to use the same manuscript to obtain a PhD in two different universities. After several conversations with Larry, we agreed that the best course of action was to return to Spain with the my academic transcripts and the manuscript of the dissertation, and try to get a Doctorate at some Spanish university. Since I studied part of my undergraduate curriculum at the University of Valencia, I contacted Isidro Ramos, which was a full professor in Physics there, and I got my doctorate in Physics there in 1982.

How would you describe the main themes projects and problems you have been interested in?

In the early 80's I continued studying the relation between complexity classes, i.e. the P vs NP problem. When I realized that solving the main problem was too big for me, I turned into other fields, as analyzing in detail problems for designing faster algorithms to get exact and approximate solutions for the problem. I also became very interested in the use of randomized methods to design and analyze algorithms and to study combinatorial structures, in particular graphs.

To give a feeling about the type of problems I have worked on, I will provide two examples: First the phase transition for the 3-SAT problem. Given n Boolean variables X and a given formula on X formed by the conjunction of m clauses, where each clause is formed by the disjunction of exactly 3 variables, or their negations, decide if there is a truth assignment $X \to \{0,1\}$ that satisfies the formula, i.e. evaluates the formula to 1. Let $r = m/n$ be the density of the given formula. Unless $P = NP$, 3-SAT is a NP-complete problem, so it could take exponential time for a deterministic algorithm to find a satisfying assignment for a given formula. In 2002, Marc Mezard, Giorgio Parisi and Roberto Zecchina using non-rigorous techniques from statistical mechanics (the replica symmetry breaking) on very large instances of 3-SAT, discovered the existence of a phase transition for 3-SAT, at a density of $r_c = 4.27$, i.e. for a small value of $\epsilon > 0$, if $r < r_c - \epsilon$, with probability $\to 1$, any 3-SAT formula is satisfiable and if $r > r_c + \epsilon$ the probability that the formula is satisfiable $\to 0$. Together with Lefteris Kirousis, Dieter Mitche and Xavier Pérez, in 2008, we provided by exact analytic methods an upper bound value to the phase transition occurrence of $r' = 4.49$, which as of today, is still the best rigorous upper-bound to the problem. I believe the value of 4.25 given by the replica method is the correct value for the 3-SAT threshold.

The second problem I would like to mention is an ongoing, non-continuous effort, lasting 18 years, which is being done together with Xavier Pérez and Nick Wormald. Infinitely many individuals are randomly placed in the infinite real line according to a Poisson point process with intensity 1. Each individual has two states, infected or healthy. Any infected individual passes the infection to any other at distance $d \geq 1$ according to a Poisson process, whose rate is $\lambda d^{-\alpha}$. Any infected individual heals at rate 1. Initially an infected individual is placed at 0, and the remaining individuals are all healthy. We want to show that, from a certain value α, there is a sufficiently small $\lambda > 0$ such that the infection process dies out with probability 1, and that there exists a large value of λ, from which the infection process lives forever with positive probability. Hopefully the paper will be finished in the early months of 2023.

I also would like to mention some problems, on which I spent a lot of time, and did not solve them. The first is a conjecture by Lenka Zdeborová and Stefan Boettcher in 2008: *In a random 3-regular graph the maximum cut size asymptotically equals the number of edges in the graph minus the minimum bisection size.* The interested reader may look at https://arxiv.org/abs/0912.4861 for the experimental evidence on the conjecture.

Another of my favorite open problems, on which I worked very hard and did not get sharp result is the following: *Find close upper and lower bounds for the expected value of the Metric Dimension on Random Geometric Graphs (RGG).* For the metric dimension problem see for example https://en.wikipedia.org/wiki/Metric_dimension_(graph_theory), and for the definitions of RGG see https://en.wikipedia.org/wiki/Random_geometric_graph.

You have played a main role in the European Association for Theoretical Computer Science (EATCS). Could you tell us about the role and achievements of EATCS?

The launching of EATCS in 1971 was an initiative due to researchers like Maurice Nivat, Mike Paterson, Arto Saloma, Grzegorz Rozenberg, Giorgio Ausiello and other mathematicians. As a by-product of the association, EATCS also initiated the following activities: The ICALP conference, with proceedings published by Springer; The Bulletin of the EATCS, which besides the usual news about the association and reports from conferences, it also includes surveys of recent TCS' topics; An associated journal, Theoretical Computer Science (TCS), published by Elsevier. The objective of EATCS was to develop a solid framework for the field of theoretical computer science in Europe, in collaboration with similar efforts taking place in the US, but in an independent way. For instance, while most of the theoretical computer science taking place in the US was mainly algorithms and computational complexity, in Europe inside the field of theoretical computer science, there was also a very strong line of research in *automata theory and formal languages*. I believe the importance in Europe of that area of research was due to the school created by two of the most "fascinating" people I ever met, Marcel-Paul Schützenberger, a Parisian Medical Doctor and mathematician, and Corrado Böhm, a mathematician in Rome. Unfortunately for me, my interaction with them was limited to a few conversations during some meetings in the 80's and 90's. But their memory is still a living legend among the theoretical computer scientists with a certain age.

In the early years of EATCS, the association had a lot of support from well known researchers in algorithms and complexity from the US, like Ron Book, Juris Hartmanis, Dick Karp, etc., which were participating in the ICALPs conferences and in weekly seminars at Dagsthul and Oberwolfach.

The frenetic evolution of information technology in the US during the 70's had a lasting influence in my academic formation. Moreover, I also believe than in the decade of the 70's and 80's, young researchers in Europe had also been greatly influenced by the growth of theoretical computer science in Europe. A good recount of those years is described in the book of Giorgio Ausiello: *The making of a new science: A personal journey through the early years of theoretical computer science*, Springer, 2018. The book gives an interesting survey of the evolution of computer science from the middle to the end of the XX century, with emphasis in the development of theoretical computer science, although among the interesting topics I liked is his personal point of view about why in the 70's Europe left to the US companies the market of building main-frame computers.

The bottom line is that EATCS was created to push the development of theoretical computer science, by creating strong collaborations of research teams from different European countries, and by creating a strong lobby for economical support at national scientific agencies, and in particular at the European Union level, which was quite effective during some years.

About my personal involvement with EATCS, I am a paying member of the association since 1980. I have been in the program committee of some ICALP's, twice as chair of the program committee. I have been in the Editorial Board of TCS for some years, and I was president of the EATCS from 1997 to 2002. In my opinion, one of the important initiatives during that period was the creation of the *EATCS Award*, which is given at each ICALP to a living scientist for the widely recognized contributions to theoretical computer science. While I was president, I could not resist the temptation of being myself the only member of the award committee, giving the three firsts awards to Richard Karp (Berkeley) in 2000, Corrado Böhm (Rome) in 2001 and Maurice Nivat (Paris) in 2002. My successor as president, Professor Mogens Nielsen (Aarhus), nominated an award committee of four scientifically well-known people, and since then the awardee has been nominated by the committee, which is renewed every year. The list of all awardees can be found in https://www.eatcs.org/index.php/eatcs-award.

On the other hand, a fact that retrospectively I'm not too happy about was that during my presidency we basically severed EATCS links with TCS, because of the high subscription prices for Elsevier's journals. At the time it loke looked a correct decision, but nowadays when I see the current situation with some *predator journals*, which taking advantage of an EU mandate for "Open Access", follow the rule *you pay and we publish fast your work, in open access*, with public institutions footing a bill of millions of euros, I have my doubts that the current situation is not doing a lot more harm, in terms of the quality of published research, that past systems, where the referee process was slow and very careful. In particular I'm worried about the image given to the young generation of researchers.

Can you share your views about the interaction between mathematics and computer science?

Computer science is a young discipline, which covers a large range of different topics. In the 70's at the US universities the teaching of computer science was split between the departments of mathematics and electrical engineering. I would refer mainly to the restricted area of algorithm design and complexity of problems, which for me includes a part of AI. To design an algorithm for solving a problem, it is necessary to understand as much as possible the problem under consideration. Trying to figure out which are the particularities of the problem that makes it difficult or easy to solve it. Notice that to understand a problem may involve modeling accurately an underlying huge dynamic network.

The history of algorithms is an old one. In the 2000's BC, the existing Babylonian tablets described some algorithms, for example to compute the square root on an integer. It is also interesting that about 200 BC the Antikythera mechanism was discovered in Greece, a small single purpose computer to predict astronomical positions and eclipses. The trend to construct single or multiple purpose machines did go on until 1936, when the mathematician Alan Turing defined the *universal computer*, computers than can simulate the work of any other computer. Returning to algorithmic issues, the first volumes of Euclides' Elements are full of geometric and arithmetic algorithms. The complexity theory developed in the 60's and 70's is deeply inspired by the advances in logic in the early XX century. The P=NP? problem is the equivalent to Hilbert's question about the existence of undecidable problems, the existence of the class NP-complete asks for the existence of unfeasible problems inside the class of decidable problems. One interesting difference between both settings is that in the case of the existence of undecidable problems, in the 1930's, Alan Turing and Alonzo Church answered the question in the positive, while, as I already mentioned, the existence of unfeasible problems (NP-Complete) remains open

I'm quite confident in saying that the overlap between mathematics and theoretical computer science is a lot larger than some mathematicians would admit. As there is also a huge overlap between statistical mechanics with algorithmic and complexity theory. For instance, in the currently very fashionable field of **deep learning algorithms**, the field has grown due to the the contributions of mathematicians and statistical physics as much as computer scientists.

I believe the following sentence from Lenka Zadeborová (a hardcore statistical physicist) reflects the synergies between the three fields: "*I enjoy erasing the boundaries between theoretical physics, mathematics and computer science*".

A good example that reflects the synergies between the three fields mentioned by Lenka is clear by looking at the contents of the textbook by Christopher Moore and Stephen Mertens: *The Nature of Computation*, Oxford UP, 2011.

Which are in your view the main open questions in computer science?

As I already said Computer Science is a wide field, and I do not feel confident to talk about the progress in the whole area, which does not mean there is not progress.

The winner of the last Abacus Medal, Mark Braverman, once said: "*Theoretical computer science is a good place for ideas from disparate intellectual areas to find a common language*". For example, studying sociological properties in networks or making epidemiological models.

I will start with my own interest in seeing an answer to the P vs NP problem. Personally I believe that the answer will be P$\neq$NP, but I know plenty of very clever researchers that believe the contrary. In any case, besides knowing the answer, what really interest me is the technique to prove the result.

Nowadays, from the practical point of view, the P vs NP problem is not too relevant. You may be interested in solving an NP-complete problem, which turns out that is easy to solve for your particular input, or you may be willing to trade accuracy for speed, and design a fast algorithm or heuristic that yields a solution that is not the optimal, but it is acceptably close to the optimal. So today, wanting an answer to the P vs NP problem is basically a matter of knowledge, not of practically being able of finding the exact solution to a particular problem. Although if the answer was that P=NP, it may help making a new effort for devising new algorithms. I do not recommend to anybody doing research for answering the P=NP? question, although I know people that are trying.

At the end of the XX century there was an unexpected event, the introduction of the internet and the web, which drastically changed many things, starting with social behavior. In the web1 many people were consuming media created by a few experts. There was a lot of hope about the usefulness of the web1, as a decentralized network, able to change and democratizing markets by using cryptocurrency, to extend many social interactions, which will push for participation and social progress. But the key of the web1 was mutual trust between users. The quick growth of the web1 gave way to the web2, with established platforms to help the users, i.e. Google, Yahoo, Facebook, etc., but also with an increasing amount of trollers. Soon the situation of the web2 was that there were a lot of users creating large amounts of free information and the big platforms were making incredible profits by commercializing that vast amount of free information. Besides that economical issue, the fact that the mutual trust is the basis of the web2's philosophy also created important problems with information veracity and reliability in the web2. That situation has promoted a movement of web users, who want to design the web3, which contrarily to the web2 will try to achieve decentralization by users designing their own systems that don't rely on trusting other users. That is exactly why I'm skeptical about the web3, I don't see masses of people programming scripts to design their own applications. But I'm sure that in the near future the web3 will co-exist with the web2 and, to develop it, theoretical research will be needed in areas as security, reliability, data structures, etc. After all, blockchain is just the starting.

The creation of Internet and in particular the different social networks has been a fantastic source of research ideas. Most of the very large social networks are what is denoted as complex networks. There has been a lot of work in modeling the dynamic growth formation of those networks. However, so far there have been few models considering the dynamics that involves vertex elimination, and the ones studied are for very simple models of networks. Another interesting area is that many of the network models and problems that have been studied are in two dimensions. But there are few results for models on high dimensional networks. That is a nice area of research. An interesting recent survey about this research area is: https://arxiv.org/abs/2203.15351.

Another interesting area of research: It is known that if P≠NP, a quantum computer will not help to solve problems that are NP-complete. However, the day a real quantum machine became available (with thousands of qubits) it will crack three important cryptographic schemes: RSA, Diffie-Hellman key exchange and Elliptic curves cryptography. One interesting line of research is the searching for new cryptographic schemes that are safe, when quantum computes become available, which in my opinion will not be soon.

Quite a bit of improvement is coming from the area of machine learning as a helper. For instance, in 2017 Roger Guimerà and Marta Sales implemented an heuristic called *symbolic regression algorithm*, where the algorithm has a library of thousands of mathematical functions. When it is fed as input with millions of data about characteristics of a certain type of cell, their algorithm searches the library for combinations of functions that interpolates the data set, and returned an equation that accurately predicts the time to cell division. Since then, variations of symbolic regression have been used to obtain equations for describing the dynamics of astronomical objects. I believe there is a large area of possible research activity in using variation of the symbolic regression method to describe dynamics systems. Notice that symbolic regression is different from a deep neural network, as symbolic regression does not "need" training, but works by combining a large set of mathematical functions.

The use of deep-learning is already becoming a very useful tool for people doing mathematics, theoretical computer science and statistical physics. Correctly trained is being used to find counterexamples, to make conjectures, to design new algorithms, to generate worst-case examples for equations, etc.. I believe that the same role Maple, Mathematica, etc. started playing 20 years ago (and are still playing), is now starting to be played by deep-learning networks. However, I will like to state my vision for the role of deep-networks in the near future. Simplifying a lot, when proving a mathematical theorem, there are two kinds of proofs, the ones that are a clever combination of refinements of existing techniques, together with quite a bit of intuition, and the ones that contain a new breakthrough idea. For me the difference of those two grossly described approaches is like between the proof that one says "that is clever, but given enough time I could have obtained the same", and the proof one says "it doesn't matter how much time I would have, I never would have come with that fantastic idea". I believe that, in a very few years, a well trained deep-learning algorithm will be able to produce some proofs of the first type described. But I'm fairly sure that we will have to wait for a long time before having a learning algorithm "creating" the second type of proofs. The day that happens, if it does, it will be a real game change for science and humanity.

How do yo see the current situation of research in mathematics and computer science in Europe? What do you think of the impact on research of the EU Framework Programs for Research and Innovation?

Since 1989 until 2016 I participated in quite a few EC Projects, and I can say that the productivity of the theoretical computer science group at UPC would have been highly diminished without those projects. For the theoretical computer science group at UPC, the participation in those EC projects produced synergies with other partners that were very important for our scientific development.

What I know about the field of information technology is that in the mid 2010's, after the enlargement of the EC membership, some of the EU Framework Programs received more political lobby from the different EC countries, which resulted in a bit of tweaking in the programs towards a more support to industry, in detriment of theory. More or less, that was kind of expected.

On the other hand, with the introduction in 2007 of the ERC grants, good theoretical researchers have more opportunities to get economical support, without having to worry with side issues, other that the quality of their research.

Personally I'm missing an open revision by the EC of the real performance of past projects, i.e. which ones they consider a big success, which ones are standard and which ones are considered a failure. Until 2000 that information was more or less open to any researcher from an EU country.

In any case, since 2018 I'm quite disconnected from the EC programs, and may be I'm missing some recent developments.

As a last question, what do you think should be the priorities for a new institute like IMTech?

To me, the immediate priority of **IMTech** should be that their members feel proud about belonging to **IMTech**, which means a bit of elitism in people's selection. That may imply to evaluate seniors researchers with more strict parameters than junior researchers. Also **IMTech** should provide useful services for young researchers, as helping to minimize their daily bureaucratic tasks, creating clusters of researchers working in the same field, advising and promoting stays in international research centers.

It will be good that **IMTech** uses social networks to spread news about seminars and courses, activities of particular researchers, visitors, awards, etc. If twitter does not crash, it could be a good starting.

I believe that in a not too long run, **IMTech** should have its own *"signature"*, i.e. an entity to apply for projects, without depending of a second party, and get the financial support to maintain their activities. Also it will be good to create a small office that helps its members to start the process of applying for a project, for example to an ERC, before it goes to the CTT. In fact that little help specific for the **IMTech** could be physically in cooperation with the CTT. Obviously, those objectives should be done in coordination with the central services in UPC. I know this last task is not easy in a university like UPC.

▷ Editorial

Jordi Guàrdia is associate professor at the Mathematics Department (DMAT) of the UPC. He has a bachelor's degree in Mathematics from the Universitat de Barcelona (UB) and a PhD from the same institution (1998), supervised by Pilar Bayer, on *Arithmetic geometry in a family of curves of genus 3* (*Geometria Aritmètica en una família de corbes de gènere 3*). Since 2001 he works in the Polytechnic School of Engineering of Vilanova i la Geltrú (EPSEVG).

His research has focused on Number Theory, mainly in arithmetic geometry and computational algebraic number theory. He is presently working in the interaction between local number theory and singularities of curves through valuation theory. He is an active member of the Seminari de Teoria de Nombres de Barcelona and has collaborated in the organization of several international conferences and workshops, being one of the founders of the "Jornadas de Teoría de Números", which this year has reached the ninth edition.

He has served as Teaching Deputy of the DMAT for eleven years, and has promoted various teaching initiatives, such as the Jornada Docent of the DMAT (Department's Teacher's Day—DTD), or the coordination of teaching materials for the different engineering schools of the UPC.

In 2022 his project *Watches, dresses and roller coasters: designing with mathematics* (*Relotges, vestits i muntanyes russes: dissenyant amb matemàtiques*), developed in the context of the subject *Mathematics for Design*" (MADI for short) has been distinguished with the 25th UPC Prize for Teaching Initiatives (Press release) and with the Vicens Vives Award for Teaching Quality.

NL. ***Congratulations for having been awarded the UPC Prize for Teaching Initiatives and the Vicens Vives Award for Teaching Quality. How do you value these recognitions in relation to your career and to the context in which the project was developed?***

Thank you so much. The truth is that the two awards have made me very happy. Since the beginning of my career I have carried out many teaching initiatives and it is nice to see the work recognised. For example, with Núria Vila and Montserrat Vela we set up, in 1993, a whole espionage role-playing game in the Arithmetic subject of the Mathematics degree at the UB. And in the UPC I have introduced computational labs for numerous subjects, but the project that has excited me the most has been the transformation of MADI, which is the one that has received these distinctions. It is a project that involves many colleagues from the EPSEVG, and that is why the awards make me more excited, because they are also a recognition of the good work that is done at the EPSEVG.

You have also been engaged with administration jobs. Could you provide details of what these commitments have been, what did you accomplish with them, and what impact have they had in your academic progression?

I have been deputy director of teaching for 11 years, first in the department MA4, which was integrated within DMAT, and after in the latter. The work at MA4 was more pleasant because it did not involve so much pure and hard management, and it allowed me to start some teaching initiatives. Then I was lucky enough to participate in the launch of the new department of Mathematics with a fantastic team. The last stage was harder, because all the time I was away with the management of the ATP (selection of part time associate teachers) and I could not deal with more qualitative aspects of teaching. Even so, we managed to organize 4 editions of the DTD. Besides, during the pandemic I did everything I could so that the teachers of the department feelt supported, both on a professional and human level. It was a tough time, but the response from the faculty was impressive. Little has been said about the work done by the educational community during the pandemic, but I think it was essential and that it helped a lot of people, and that it deserves much more recognition than it got.

As for my progression, it's obvious that all this work has had a negative impact on my research, which has slowed down, and I'm sorry for that. But I don't regret it, because I always experienced it as a service to the community and, moreover, overall this has all been a very enriching experience.

Although you have been assigned to the EPSEVG, you have also taught at the FME. Besides, you have brought a helping hand in many of the FME's initiatives. How would you summarize these undertakings and the significance they have had for you?

It is a blessing to be able to impart teaching at the FME. Students and faculty have great enthusiasm and a motivation that is commendable. You have the possibility to collaborate with people from other backgrounds and you learn a lot. For example, last year I collaborated with the Imaginary exhibition, as a guide for high school groups and it was very interesting at all levels. The possibility of working in two centers with such different approaches as the EPSEVG and the FME provides a more global perspective on the teaching of mathematics.

By the way, the FME has been celebrating its 30th birthday. How do you envision its evolution in the coming years?

The FME is a center that has always worked very well since its birth, it is a ship that is sailing at full sail and I am sure that it will continue to do so. In the coming years, we will have to face at least two important problems: the aging of the workforce and the gender bias. It is always said that the aging of the workforce seriously affects research, but it also seriously affects teaching, and we must try to reduce this effect. Regarding the issue of gender, the contribution of women to the world is a luxury that we cannot continue to do without; but the percentage of female students and faculty at the FME is very low: Together we must promote initiatives to reverse this situation as soon as possible.

We will also have to face the current great demand for mathematicians. Today's society needs more mathematitians than we can promote in the faculties, and we should not evade our social commitment in that respect. It will be necessary to study the possibility of increasing admissions to the FME, but in a considerate way, so that it does not over stress the system and guarantees the quality of the graduates.

You have taught many courses on a variety of mathematical subjects and in different schools. What motivations have concurred in your choices? When facing a new course, what principles do you follow in its design and implementation? Can the methodology of your prized project be adapted to other scenarios?

Yes, I think I have taken more than 40 different subjects, and this has allowed me to greatly expand my mathematical training. I have always tried to do subjects that attract me and that bring me new knowledge. When I get involved in a subject, I try to do it gradually: if possible, I start by doing only problem or laboratory classes, to get to know the content well; then I take theory groups and, when I have gained confidence, I see what contributions I can make to the syllabus or the teaching methodology. In the case of MADI, in which the awarded project has been developed, I have been involved since 2013.

The award-winning project has several axes, such as the use of social networks to get closer to students. But from a mathematical point of view, the most important thing is that we have given a very applied approach to the subject. For example, when we explain Frenet's frame, we don't worry too much about students learning to calculate it by hand, a computer will do that for them when they need it, but instead, we make them design a road or a railway where its use is inevitable. In addition, in all the tasks that we set for our students, who are from the Industrial Design degree, they can develop their creativity, so that they get more involved and produce spectacular work. Perhaps not all the proposals in the subject can be extrapolated to other contexts, but certainly many are, and I will give you another example: to study the usual parameterization of the sphere in spherical coordinates, we have our students draw, with Geogebra,a typical colored basketball.

Good teaching stems from good research (in a broad sense). What have been the main endeavors of your investigations ever since your PhD thesis? What lessons from these experiences have influenced your teaching? Vice versa, are there problems emerged from a teaching context that have morphed into good mathematical research problems?

For a university professor, being up-to-date in his area (at least) is essential. My research has allowed me to incorporate many things into my teaching. I have worked a lot in computational number theory, which has many applications to basic problems, which can be perfectly explained in class. Beyond the usual examples of cryptography, there are surprising results that fascinate students. For example, everyone takes it for granted that we can factor any polynomial with rational coefficients, because we are taught to do it from a young age, but only with prepared examples. The reality, however, is that it is a very hard computational problem, and in fact the first complete algorithm to solve it is only 20 years old! Today's students were hardly born when this algorithm was formulated and when I explain it to them they are very surprised and interested in the subject.

In the reverse direction, part of my initial research arises from a classic Algebra problem that I have explained many times: the solvability of polynomial equations. In my early work I gave generic formulas for the roots of a polynomial of any degree in terms of theta functions.

In any event, what is the main focus of your current research efforts? What would you like to achieve in the near future?

For many years I have collaborated with Professor Enric Nart (UAB) in the development and applications of the so-called *Montes algorithm*, which is becoming a basic pillar of computational algebraic number theory. Lately we have focused more on the geometric side of the algorithm and, together with Maria Alberich and Joaquim Roé, we are applying it to valuation theory. At the same time, I am working in the field of monogenic extensions and collaborating in the construction of tables of fields of numbers. These may appear to be superficially distant subjects, but there are clear connections between them, and I would like to be able to exploit these connections to advance all three subjects. ▷ Editorial

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Josep Maria Rossell and Núria Salán have been distinguished *ex-aequo* with the 2022 UPC Prizes for Quality in University Teaching (25th edition) for their teaching career. These prizes are awarded by the Social Council of the UPC. J. M. Rossell has also been distinguished with the Vicens Vives Award for Teaching Quality.

Professor J. M. Rossell is affiliated with the UPC/DMAT and he works in the Manresa School of Engineering (EPSEM). The jury has highlighted the vocation of helping and accompanying the students, as well as the persistence over the years in the teaching quality, as witnessed by the results and the satisfaction ratings of the mathematics subjects he has taught, in various courses of different specialties, throughout his career. The jury has especially highlighted Rossell's work in teaching mathematics in the first years of university studies, with very large groups of students from varying academic backgrounds and very heterogeneous levels of knowledge. Likewise, his spirit of collaboration with the rest of the teaching staff has been valued.

Professor N. Salán is affiliated with the UPC/CEM and her teaching is assigned in the School of Industrial, Aerospace and Audiovisual Engineering of Terrassa (ESEIAAT). The jury has highlighted her tireless endeavors for finding creative ways of teaching in the classroom and in the laboratory (in content design and learning activities), and the effort to bring the friendliest side of engineering teaching closer to the society. With this award, the Social Council wanted to highlight the teaching quality of Núria Salán, strengthened by the explicit support of the student community through the ESEIAAT Student Delegation, which highlighted the proximity, passion and the innovative teaching approach of Salán, as well as her involvement in helping and the flexibility to adapt to the needs of the groups of students. Likewise, it has been valued that her innate curiosity, transferred to the students, has facilitated their involvement in work groups on outstanding projects.

NL. ***We are proud that the quality of your teaching career has been recognized with the awards mentioned above. How have you experienced getting these distinctions? What significance do they have for you?***

NS. An acknowledgment is always evidence that what you have done has been minimally correct. And this is very encouraging. Student evaluations are the main indicator, since I have always proposed methodological changes to help them in their learning process. The awards also help to give visibility to what you have done, to your proposals and initiatives, so that someone may be interested in what you do and consider incorporating, in their teaching activity, some methodology that they may find useful.

JMR. I have to say that it has been an enormous satisfaction to have received these two awards, given the large number of teachers who apply. It is a recognition of a long academic career, of almost 38 years, at the UPC. In addition, I believe that for a small center such as the Manresa Campus, these distinctions still have added value, as they show that their teachers can also do high-quality work.

How has your teaching philosophy evolved along your academic career?

JMR. Of course, the teaching activity has changed over the years. When I joined the university as a professor in 1985, we still didn't have computers. At that time, all classes were theoretical and there was no software for subjects such as calculus, algebra or statistics, among others. As the classrooms were equipped with computer equipment, the classes began to have a more practical, more applied aspect. In addition, the use of digital platforms provided another way to contact the students, certainly interesting, but sometimes a little too impersonal for my taste. Despite everything, my attitude as a teacher has always been the same when teaching the courses, always trying to be as close as possible to the students.

NS. It is not necessary that all my students are "Marie Curie", they can be perfectly "Marie Pérez". Let me explain: I often find that the interpretation of excellence, in teaching, is linked to complex academic content, very dense syllabi and very strict processes for student evaluation. I totally disagree. Over the years, I have been loosening the severity of the subject programs, I have tried to reduce the rote contents to insist on contents that link contents, favoring the connections between contents and avoiding isolated topics.

Can you share with us the secrets of you success? What advice would you give to colleagues that struggle to improve their teaching of mathematics courses?

NS. Over the years, I have incorporated stories and anecdotes, linked to the theoretical content, because if they remember the anecdote they can at least remember some part of the content. And also because the stories and anecdotes are, in reality, small episodes of learning. I have opted for the debate and for the projects that must be carried out linked to the contents of the subject, and I have introduced role-playing games (where the students adopt the role of a junior engineer and I am a very demanding). This has contributed, I think, to reducing the initial stiffness of a subject like mine. My professional expertise is linked to failure mechanics and forensic engineering studies. And that is very sexy. I have learned to explain matter in reverse: first I expose the disaster and chaos generated by an unexpected historical failure, and I go back, looking for reasons why it could have happened, what could have gone wrong, what we didn't see, and so I get to the subject matter I wanted to explain. It's more work to do it this way than in the conventional format, but I think it's more effective. And more pleasant. If, in a class, the students stop taking notes and listen, I understand that it is good, because the notes they can take are already in books that someone has written and that, generally, their contents has been revised and has no errors, while the notes you may take may always have some inaccuracy. I always suggest that they read the contents prior to the class (I give them several options, links to MOOCs and YouTube videos, so that each one chooses the most suitable way) and we hold debate or *agora* type sessions, where I present examples, case studies, success stories and, above all, failure cases, because you learn a lot from failures. I would tell the young colleagues to take advantage of a good collection of MOOC sessions and/or YouTube videos with content pills about their subjects and to consider using them as teaching material, so that they will have time to devote themselves to explaining something else or to proceed more calmly.

JMR. I have always claimed face-to-face teaching, classes with blackboard and chalk. I am not in favor of uploading an avalanche of material and believing that, simply by giving a lot of information, the students are better informed. A teacher must be able to assess on the spot if the students are following the explanations correctly and, if necessary, must modify the pace and level of the lessons so that everyone can take in the content. It goes without saying that the classes must be taught in a dynamic and enthusiastic way, but inserting short interruptions when the subject is cumbersome and taking advantage of these moments to explain some mathematical curiosities that, on the other hand, serve to increase knowledge of the discipline. The stereotype that Mathematics is difficult and boring must be eradicated and that is the job of the teacher. It is difficult, however, to give advice to other fellow teachers, as the way you act when teaching a class is somewhat innate. Training students is not easy and requires a lot of effort, dedication and above all a good dose of enthusiasm.

What are the main lines and accomplishments of your research?

JMR. My research topic focuses on Applied Mathematics, more specifically on the theory of automatic control of systems. For many years, I have worked on the study of vibration control of flexible structures, mainly in buildings, bridges, marine wind turbines and vehicle suspensions, both from a theoretical point of view and experimental simulations. For the more experimental part, I am using two Stewart platforms, a type of parallel robot that allows simulations of the movement of an object or a structure with six degrees of freedom. It is worth saying that these mechanisms were designed and built by our research group and that they have expanded our research objectives.

NS. My research is linked to stainless steels, the development of new grades of hard metal, the design of new types of coating for tools (mainly) and manufacturing processes of composite materials. The content developed in research allows me to give current examples, identify cases of improvement, and be able to expose cases of failure (processes that have not worked, initiatives that have not gone as well as expected, etc.). I also do research in learning methodologies, linked to technical-technological studies, although this research counts little for accreditation processes. But I keep doing it. A third area of research that I started a few years ago is related to gender issues: Why do we not yet have enough girls in engineering studies? When do we lose them? What makes engineering not attractive enough to boys and girls alike? This research has given me a lot of personal satisfaction and has helped me to identify characters (inventors and researchers) who have a connection with the contents of my subject, so I put examples of the type "we can enjoy this thanks to xxx" and say the name and surname of the woman who made it possible.

How do you manage the binomial research/teaching?

NS. As long as you're learning, you can keep teaching. I hope to never stop learning, because teaching evolves if the content to be transmitted, and the transmission methodologies, do likewise.

JMR. I couldn't say whether I prefer teaching or research, but what is clear to me is that I am passionate about both. If I had to report on how do I split academic time, I think my dedication is about 50% for each. It is true that I have always conceived of teaching as my profession, I feel like a vocational teacher. However, the research has given me a lot of satisfaction since seeing articles published in high-profile journals or participating as a speaker at conferences around the world is priceless. Being able to combine the two academic sides I think should be a duty of any university professor.

What are the advantages or disadvantages of working in a 'peripheral' UPC campus?

JMR. The advantages of working on a peripheral campus are manifold. First of all, if we are talking about a not very big campus like Manresa, it allows you to meet and connect with all the people around you in a personal and close way. In addition, the classes are usually smaller, which means having a closer relationship with the students and being able to attend to them more individually. However, it also has its drawbacks, as being far from the heart of the University means that things are sometimes lost, for example, the opportunity to recruit PhD students, to make important decisions that affect academic life or to participate in activities that are usually concentrated on the larger campuses.

NS. The UPC starts at Garraf and ends at Bages. Between Vilanova and Manresa we have a large campus with locations scattered throughout the territory. Outside of this territory it would be appropriate to say that we are in a peripheral area... If the question is what advantages or disadvantages do I find working outside of Barcelona, then the answer would be different: "non-Barcelona" campuses generally allow a different interrelationship between students and teachers. The relationship with the environment, with the territory, is also experienced differently, since you are part of the daily life of your city, and when you leave the school gates, you are in a city where there are schools, where people who look at the school, on their way to wherever, and explain "here studied...". I recently attended an event at the Barcelona City Council that wanted to raise initiatives to increase the presence of women in minority fields, such as technology, and they had not invited anyone from the UPC. I was there, but as president of the Catalan Society of Technology. They had summoned someone from the UPF and someone from the UB, but nobody from the UPC. I had the feeling that the the administration's ignorance of the reality of its academic-training environment is much greater in a large city than in a non-Barcelona locality, where the councils always, always, know their UPC schools . And they count on them for countless initiatives. Disadvantages: having to travel to Barcelona for meetings that are "centralized", although, lately, we have learned to incorporate face-to-face-virtual duality and this has greatly contributed to encouraging participation in meetings and activities, because they have eliminated the time lost in travel.

How do you envisage your academic activities in the next years?

NS. I'm afraid I'm pretty close to retirement, so I'd like to be able to pass on the material I have to anyone who would be willing to use it. I don't want to say that everything I have (books, notes, projects, slides, videos, etc.) can be readily used by others; what I mean is that if any of those materials can be helpful for someone, I'll be happy to make them available to them. Naturally, I will keep the rest. I would like to participate in educational projects with young teachers, to learn about their initiatives and to be able to help, if necessary, with what I have experienced during my years as a teacher. I would like to be able to create a unique book/text/material for my subject, and when I say "unique" I mean that it has been designed and developed together with other materials' departments of other universities around the territory, to promote the mobility of our students, both during their academic and professional lives. Wow, I have more projects and work in my mind than years ahead...

JMR. In terms of teaching, I want to keep adapting to the new educational models that are coming to us, in order to transmit the knowledge of Mathematics to the students in the best possible way. On the topic of research, I would like to continue leading the research group I currently have, obtaining new scientific results, attending presentations and international forums and leaving my research team ready for my future release as principal researcher with a bag full of goals and ideas to develop.

N. Salán, *you are deputy director for institutional promotion of ESEIAAT. Could you describe your vision about that service and the main activities which you are pursuing to enact it?*

This position allows me to be in daily contact with pre-university educational centers. My main task is the promotion of the studies: Bring the reality of engineering closer to young people in primary school, compulsory secondary education, and high school by presenting all the opportunities that our curricula offer; to share everything that can be done with a technological training with teachers' cloisters, with the associations of students' families and friends, and with a diversity of students. In addition, my work as deputy director for institutional promotion involves making sure that all the school's students feel at home, that no question remains unanswered, no consultation unresolved... Everyone has my contact (mail and Whatsapp) and they know they can contact me for any doubt, consultation or concern. This proximity is part of my job, and I find it very necessary.

J. M. Rossell, *you have also been awarded the Vicens Vives distinction of the Generalitat de Catalunya (Catalan Government). Could you comment about the significance of this recognition for your career?*

The distinction Jaume Vicens Vives 2022 has been the greatest recognition obtained throughout my academic life. It should be noted that, in each edition, the winners of the awards for the best teaching career, previously awarded by the social councils of each of the public and private universities in Catalonia, take part. It is therefore an enormous personal pride and satisfaction, and I think also for the Manresa campus, to have obtained this award. ▷ Editorial

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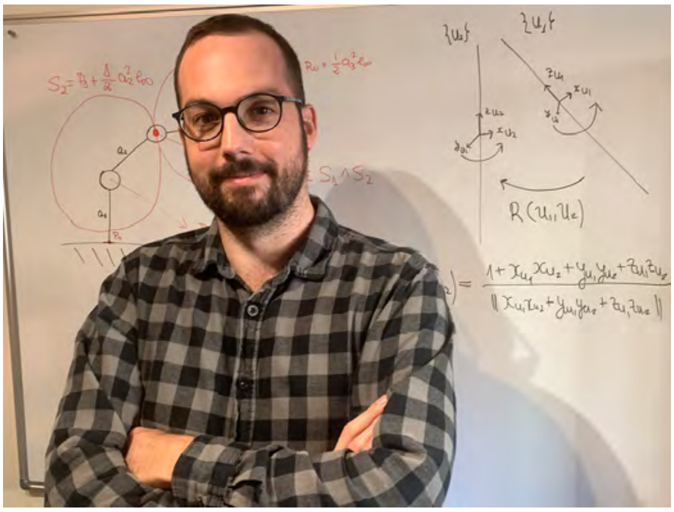

Isiah Zaplana (Cartagena, 1989) completed his bachelor's and master's studies in Mathematics in 2013 (Universidad de Murcia and Universitat Politècnica de Valencia). Then, he was granted a national fellowship to pursue his PhD's studies at the Institute of Industrial and Control Engineering (IOC) of the Universitat Politècnica de Catalunya (UPC), where his PhD project focused on the development of several geometric and numerical methods to deal with two classical problems in robot kinematics, namely the inverse kinematics and the singularity identification problem.

After finishing his PhD in 2018, he moved to Italy to work as a postdoctoral researcher at the Advanced Robotics Department of the Italian Institute of Technology (IIT). There, he complemented his background by learning about industrial robotics, automation and applied machine learning. From 2021 to 2022, he worked as a postdoctoral researcher at the Department of Mechanical Engineering of the KU Leuven (Belgium), where he used and applied all his previous background (especially applied geometry, robotics and machine learning) to the development of robotic strategies for the optimal disassembly of electric and electronic devices. In particular, he has geometrically characterized both the object to be disassembled and the obstacles present in the workspace to compute an optimal free-from-obstacles trajectory for the robot to execute its task. Finally, since December 2022, he is a Maria Zambrano Postdoctoral Fellow at the IOC in the UPC. His research interests lie in the intersection between applied mathematics and robot kinematics. In particular, he has explored novel ways of using geometric, algebraic and numerical tools to deal with classical problems in robotics. One of these tools is (conformal) geometric algebra, a mathematical framework where both the geometric objects and Euclidean rigid transformations can be encoded as elements of the algebra. Complementary, he is also very interested in numerical optimization methods (especially meta-heuristics approaches) and their application in robotics (e.g., optimal parameters computation for automated processing lines and optimal path-planning calculation). In addition, he has also started to gain interest lately in the applications of machine learning to robot kinematics, especially in combination with algebraic geometry and conformal geometric algebra.

NL. *Congratulations for your María Zambrano postdoctoral fellowship. But let us begin at the beginning. What led you to choose Mathematics when you enrolled at the Universidad de Murcia? What memories would you like to share from that period?*

During high school, I was really into mathematics. In fact, I remember that when studying at home, I was looking for any excuse to stop studying the other subjects and do other exercises in the maths book (even those that the teacher did not ask). The best ones were those at the end of each lesson, which were the most challenging and tricky of all. Then, when in the third year of high school (around 14 years old), I had a math teacher with a passion and love for mathematics that was, not only contagious but also inspiring. Then, I talked to her because I wanted to know what do you need to do to study maths at the university and she explained to me that mathematics has nothing to do with what I was studying and that it is a far more abstract (but also beautiful) subject than the simple arithmetic we studied at class. That was definitively the straw that broke the camel's back. I thought, I really want to study this. And here I am. Regarding my years as a maths student, I have two memories in particular that always come back to me when I remember those days. The first was the shock of discovering what Mathematics really is. On my first day, which I will never forget, there was a sequence of professors presenting theorem after theorem, and it was a bit overwhelming. A feeling that persisted, I must say, during the first months. The second memory is that it was pretty tough to choose the optional subjects for the last course (in the old plan consisting of five years, the last had only optional subjects). The reason was that I wanted to study them all ... geometry subjects, analysis subjects, and algebra subjects. I liked so much what I learned during the first years that I could not decide on a particular area and

I ended up doing a mixture of everything haha.

Then you moved to the Universitat Politècnica de Valencia to pursue a master's degree in Mathematics. What features of that program were decisive for you?

The master program was called INVESTMAT, which stands for Master en Investigación Matemática (Master in Mathematical Research in English). It was taught by both the Universitat of Valencia and the Universitat Politècnica de Valencia, i.e., by the Department of Mathematics and the Department of Applied Mathematics. This put a distinctive imprint on the master, which was a combination of pure and applied mathematics. My contact with applied mathematics was very little at that time as the Mathematics studies at the Universidad de Murcia were more theoretical. Therefore, this master's degree was a perfect opportunity to continue learning pure mathematics and, at the same time, introduce myself to the ocean of applied mathematics, a bag into which pretty much everything can enter. I was not aware of the potential of mathematics in other fields such as, for instance, engineering, biomedicine, art, etc. It was a very interesting experience and it turned out to be decisive for my next career steps.

Your PhD work was developed at the IOC of the UPC, an engineering institute. How do you value your previous education in mathematics in relation to the topics that were the focus of your PhD?

As I was telling you before, during my master, I learned and fell in love with the applied side of mathematics. Therefore, when a PhD position on theoretical aspects of robotics was offered, I right away applied for it. It was funded by the Economy Affairs Ministry, i.e., was a national PhD fellowship. It was my first contact with robotics, and my first impressions were soon confirmed: robotics lies on a large mathematical ground. From dynamical systems to model and control the dynamics of the robot to non-linear geometry and algebra to model its kinematics and movements in Euclidean Space. Although all the research lines being developed at that moment were a bit more applied, my supervisor gave me the freedom to choose and develop my own project (always in the framework of robotics). I chose a topic focused on the development of geometric solutions to two fundamental problems in robot kinematics, namely the inverse kinematics problem and the singularity identification problem. To address both problems, my education in mathematics was decisive. First, like many mathematicians, I have a very analytical mind. I always take the problem, analyze it carefully, divide it (if possible) into simpler problems and I attack each of the sub-problems independently but keeping in mind the big picture. In addition, because of the chosen topic, my background in geometry and algebra has turned out to be very helpful in understanding the geometric component that underlies robot kinematics and how to use all the geometric and algebraic tools at hand to attack the two problems I chose to work with during my PhD.

During your PhD research, were you already aware of the potential of geometric calculi for the problems you were interested in, particularly in robotics?

Not at the beginning. One year after I started my PhD, I attended a seminar on geometric methods for vision. The speaker, Carlos Lopez Franco, worked on how conformal geometric algebra can be used to model and solve geometric problems in vision. During his presentation, I was amazed by this new language to model Euclidean and conformal geometry. It is not only computationally friendlier, but also more intuitive, so you can also develop geometric solutions that are unfeasible with classical or analytical Euclidean geometry. Its potential is unlimited. For instance, an open problem in robot kinematics was the definition of a distance function able to evaluate the distance of any robotic configuration to a singular configuration (a configuration where the motion of the robot is impossible in at least one spatial direction (either translate along that direction or rotate around it)). The study of singularities is a very important problem in robot kinematics as these special configurations greatly affect how the robot moves in space. Therefore, having a distance function able to measure how far/close you are from a singularity allows for the definition of control or planning strategies to pass arbitrarily close but without being affected by them. However, because of several reasons (the details are maybe too technical to be discussed here), this is not possible with classical geometric tools. But it is possible with geometric calculi thanks to the rotors, elements that in the conformal model of the Euclidean space, encode all conformal transformations, in particular, rotations and translations. This is to me a very beautiful result!

What influence did your stay in Cambridge, and in particular your contact with Joan Lasenby and her school, have in your research agenda?

Joan is absolutely great. She is a great scientist and an even better person. Her passion for science is contagious. During my stay in Cambridge, I re-learned geometric calculi, advanced a lot in my research (I completed the results for half of my PhD thesis in the three months I was there), and met a lot of people in the field with whom I am still collaborating. I always remember her telling me how important rotors are, that they constitute one of the key elements of geometric calculi and definitively a tool to use anytime you face a complex geometric problem. She was, of course, right. As I have said before, rotors were the key to solving a very complex problem in robot kinematics that was still open. In addition, our collaboration did not end after my research stay in Cambrige, but it has continued until today, as we are still working together.

You have been a postdoctoral researcher in Genoa and in Leuven. These must have been important steps in your career, and thus we would like to know your views on each of these stays and also on comparing the two research atmospheres.

I have complemented my background in both places. For instance, in Italy, I learned a lot about automated robotic processing/assembly lines. In particular, how to model and optimize them to increase their efficiency and avoid downtimes (something that nowadays is called digital twins). It was also there that I started to use artificial intelligence and, in particular, computational intelligence, to optimize complex systems in a more efficient way. Later, in Belgium, I learned a lot about the integration of deep-learning computer vision with robotics. In particular, how the integration of both yields more robust and efficient robotic systems for the handling of end-of-life electric and electronic equipment. In both postdoctoral experiences, I have been able to continue working on the fundamental topics I started during my PhD and, at the same time, learn new things that have been super valuable for me as a researcher. In addition, in Leuven, I also learned other types of skills, such as how to write project proposals and apply for funding, how to coach PhD students, etc. Regarding the research atmospheres, they were surprisingly similar. There were differences of course. In Italy, as well as in Spain, funding is always very little and, in my opinion, insufficient for developing good research (especially if it is applied research). However, in Belgium, there are many opportunities to get funding within the Belgian government and it is very common to have between 5 to 10 PhD students and several postdocs per group, something that I have never seen in Italy or Spain, where the research groups are in general more humble. But apart from that, research-wise I will say that there are not that many differences and that, nowadays, the way of doing research is pretty similar around the world.

Could you please describe your recent research achievements, including your renewed collaboration with Joan Lasenby's school?

As I have said before, one of the last achievements relates to the definition of a distance function to measure how close the robot is to a singularity and that was achieved thanks to the use of rotors to model rigid transformations between the joint axes of the robot. This was done in collaboration with Joan and one of her PhD students, Hugo Hadfield. In addition, also together with Joan and Hugo, last Spring I published another journal paper where we used conformal geometric algebra to deal with the inverse kinematics of both redundant and non-redundant serial robots. Inverse kinematics is the problem consisting of finding all the robot configurations that make the end terminal of the robot have a given position and orientation in space. The majority of proposed solutions (especially for redundant robots) are numerical, i.e., they approximate only one of those configurations. The solution we proposed allowed the computation of all the solutions in closed-form, i.e., as functions in terms of the given position and orientation of the end terminal of the robot. Finally, during the last two editions (2020 and 2023), I have been organizing, together with Sebastià Xambó-Descamps (1st and 2nd edition), Pierre Dechant (2nd edition) and Yang-Hui He (2nd edition), a Session in the International Conference on Clifford Algebras and Their Applications in Mathematical Physics (ICCA) on machine and deep learning with geometric calculi, where we stimulate the interest on the research community for the intersection between artificial intelligence (in all its aspects) with geometric calculi and the vast ocean of possibilities this intersection makes possible.

Now your fellowship at IOC looks as a new horizon for your academic life and so we would like to ask you about your general goals in that respect and about the specific goals that will guide your research.

My next step is trying to get a position as an assistant professor here at the UPC and start the way towards a permanent position as a professor within the university. In the meantime, I will continue with my research lines, namely the development of geometric solutions based on geometric calculi for open problems in robot kinematics. While in my PhD, I worked exclusively with serial robots, i.e., robots that can be kinematically modeled as an open kinematic chain, now I will also work with closed kinematic chains, which are kinematically more complicated as there are additional mechanical constraints to the movement whose modeling and treatment are not easy. Finally, due to its similarities with robot kinematics and dynamics, and in collaboration with Joan and other colleagues, I will see how all these strategies can be applied in molecular dynamics. In particular, how to geometrically characterize them and study the behavior and motion of especially complex molecules, such as proteins.

Although a good part of your work lies within engineering disciplines, your early education was fundamentally mathematical, and we wonder how do you see this discipline today and what role do you see for it in the scientific and engineering landscape of today and tomorrow.

That is a very interesting question! In fact, all my past experiences (in the IOC during my PhD, and in IIT and KU Leuven during my postdocs) have confirmed what I believed when I finished my master, that is, that mathematics has always been and is becoming again a fundamental part of modern science and engineering. I have seen how more and more engineering research groups become multidisciplinary research teams in order to join forces from different backgrounds and, thus, attack more complex and challenging problems. In fact, mathematicians have become a very valuable asset in every research group. This is a trend that I strongly believe will continue in the coming years, and that, personally, I consider pretty exciting! ▷ Editorial

Research focus

Darmon's conjecture on Stark-Heegner points by Victor Rotger (DMAT, **IMTech**).
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Motivation: the conjecture of Birch and Swinnerton-Dyer

In order to celebrate mathematics in the new millennium, the Clay Mathematics Institute proposed seven challenges and one of these is the conjecture of Birch and Swinnerton-Dyer (BSD), widely open since the sixties. A large portion of my research is devoted to the conjecture of Birch and Swinnerton-Dyer on elliptic curves over number fields and their generalizations by Bloch and Kato (BK) to arbitrary motives associated to higher-dimensional algebraic varieties over global fields, and here we report on recent progress in this direction due to H. Darmon and myself in [DR3], [DR4].

An elliptic curve over a field K is a smooth, projective algebraic curve of genus one over K, on which there is a specified rational point O. When the characteristic of K is different from 2 and 3, they are usually exhibited by means of an affine Weierstrass equation

$$E : y^2 = x^3 + Ax + B,$$

where A, B are elements in K such that the discriminant $4A^3 - 27B^2$ does not vanish.

The geometry and arithmetic of E is pretty well-understood when the ground field K is the field of real or complex numbers, or a finite field. However, when K is a number field, that is to say, a finite extension of the field of rational numbers, the arithmetic of E becomes highly mysterious and is the object of many intriguing open questions. BSD in concerned with the (seemingly innocuous) problem of determining the whole set $E(K)$ of points on E whose coordinates lie in K. To convey the difficulty of the problem, it is best to illustrate it with an example. Take the elliptic curve

$$E : y^2 = x^3 + 7823$$

over the field of rational numbers. Finding a non-trivial solution to this equation over $\mathbb{Q}$ is a highly non-trivial problem: few years ago, Stoll employed a 4-descent method to show that the coordinates of the simplest solution are

$$x = 2263582143321421502100209233517777/119816734100955612,$$

$$y = 186398152584623305624837551485596770028144776655756/119816734100955613.$$

Needless to say, the task becomes even more daunting when one faces the problem of proving general statements about the set of solutions of a whole class of equations over a wide variety of number fields. Elliptic curves are distinguished by the fact that they are equipped with a rich algebraic structure: a composition law that makes it possible to generate new solutions from already given ones. This law is at the heart of the many practical applications of elliptic curves to coding and public-key cryptography.

When K is a number field, $E(K)$ is a finitely generated abelian group, called the Mordell-Weil group of E/K, and its rank $r = r(E/K)$ is the most important and mysterious invariant of the global arithmetic of E/K.

Besides, associated with the elliptic curve there is a holomorphic function $L(E/K, s)$ commonly referred to as the L-series or zeta function of E/K. It is a complex-analytic function defined by an infinite product ranging over the set of prime ideals of the ring of integers of K which encodes the number of solutions of E over the associated finite residue fields. While this product only converges absolutely on the right-half plane $\mathrm{Re}(s) > 3/2$, it is conjectured to extend to an entire function on the whole complex plane and to satisfy a functional equation whose center of symmetry is the point $s = 1$. Such expectations are known to hold in many cases, for instance when K is the field of rational numbers, thanks to Wiles' celebrated modularity theorem.

The Conjecture of Birch and Swinnerton-Dyer states that the rank of the Mordell-Weil group $E(K)$ should be equal to the order of vanishing at $s = 1$ of $L(E/K, s)$. This conjecture may be interpreted as an analogue for elliptic curves of the analytic class number formula for number fields, and was proposed by Brian Birch and Peter Swinnerton-Dyer in the sixties, who verified it numerically in a number of examples. A more refined version of the conjecture formulates a precise recipe for the first non-vanishing coefficient in the Taylor expansion of $L(E/K, s)$ about $s = 1$ in terms of several local and global arithmetic invariants of E, including the cardinal of its Tate-Shafarevic group, which at present it is not even known to be finite.

BSD stands as the tip of the iceberg formed by the vast conjectural program of Beilinson, Bloch and Kato, and all the attempts taken so far to proving it exploit the deep connections between Shimura varieties, Galois representations and automorphic forms. Hence the conjecture can actually be stated in a much more general context, including the twist of E by irreducible Artin–Galois representations of the absolute Galois group G_K of K. The generalization of BSD formulated by Bloch and Kato [BK] applies to arbitrary motives arising from higher-dimensional varieties.

State of the art

In 1976, Coates and Wiles [CW] came up with the first breakthrough on BSD. Their result became ten years later a particular case of the ground-breaking theorem of Gross, Zagier and Kolyvagin [GZ], [Ko], which was in turn generalized by Zhang by means of a similar but more modern and robust approach. The combination of their results allowed to prove BSD for the base change of all *modular* elliptic curves $E/\mathbb{Q}$ to an anticyclotomic abelian extension H/K of an imaginary quadratic field K, provided the order of vanishing of each of the factors of $L(E/H, s)$ is at most 1. The celebrated work by Wiles and his collaborators on Fermat's Last Theorem showed that all elliptic curves over $\mathbb{Q}$ are modular, and hence one could deduce the following statement from the above results:

Theorem 1 (Coates, Gross, Kolyvagin, Wiles, Zagier, Zhang)**.** *Let $E/\mathbb{Q}$ be an elliptic curve.*
BSD_0: *If $L(E/\mathbb{Q}, 1) \neq 0$ then $E(\mathbb{Q})$ is torsion.*
BSD_1: *If $L(E/\mathbb{Q}, 1) = 0$ and $L'(E/\mathbb{Q}, 1) \neq 0$ then $E(\mathbb{Q})$ has rank 1.*

The above claims were proved in all cases by exploiting a suitable Euler system. In the literature one encounters different conceptions of what an Euler system is, ranging from the very systematic approach of Rubin to the more flexible point of view adopted in [BCDDPR]. In all known instances, an Euler system consists of a norm-compatible collection of global Galois cohomology classes arising from geometry, whose local components are related to special values of p-adic L-functions. Even if one takes the most flexible definition of what a decent Euler system should be, there are very few known examples of them. Coates and Wiles invoked the Euler system of elliptic units for proving statement BSD_0 above in the setting where E has complex multiplication. Gross, Zagier, Kolyvagin and Zhang exploited instead the Euler system of Heegner points to prove BSD_0 and BSD_1 for arbitrary elliptic curves.

Nekovar generalized these results to the motive associated by Scholl to arbitrary eigenforms of higher weight, proving this way a non-trivial instance of Bloch-Kato's (BK) conjectures. Soon after [GZ] and [Ko] appeared in press, Kato [Ka] surprised again the world with another astonishing result: he managed to prove BSD for twists of elliptic curves by Dirichlet characters of arbitrary order, provided the L-series does not vanish at $s = 1$. In particular, BSD follows for the base-change of E to any abelian extension $F/\mathbb{Q}$, provided $L(E/F, 1) \neq 0$.

While the progress achieved at the end of the eighties invited to some optimism, the whole number-theory community was at a loss for more than twenty years at finding new ideas for tackling BSD. The drought ended a few years ago, when at least three ground-breaking and completely different approaches saw the light around five years ago:

(1) the work of the Fields medallist M. Bhargava, in collaboration with various mathematicians (cf. [Bh] and references therein),

(2) the work of C. Skinner and E. Urban ([SU] and their subsequent publications) and

(3) my own work with M. Bertolini, H. Darmon and A. Lauder: cf. [BDR], [DR1], [DR2], [DLR].

The three points of view are complementary and cross-fertile. The counting techniques pioneered by Bhargava allow to prove results of statistic nature on the average size (resp. rank) of the Selmer group (resp. Mordell-Weil) group of an elliptic curve. Besides, Skinner and Urban exploit families of automorphic forms on unitary groups to prove the Iwasawa main conjecture for $\mathrm{GL}(2)$, which allow them to prove converses of the theorems of Gross, Zagier and Kolyvagin. Finally, my work with Darmon introduces a new Euler system: the system of diagonal cycles varying over a triplet of Hida families (cf. [DR1] and [DR2]). It allowed us to prove many new instances of BSD in rank 0 for a large family of number fields which was beyond the scope of previous techniques. Specially striking is the case of anticyclotomic abelian extensions of real quadratic fields, as this was the ideal application of Darmon's still unproved exciting conjectures [D] announced at the ICM in Madrid 2006.

Darmon's conjecture on Stark-Heegner points

As mentioned already, at the end of the previous century the only known contribution to BSD were the results of Coates-Wiles, Gross-Zagier-Kolyvagin and Kato. In particular BSD was a mystery already in the following simple setting, which was considered to lie just beyond the techniques that were available at the time:

Let $E/\mathbb{Q}$ be an elliptic curve of prime conductor p over the field of rational numbers. Let K be a real quadratic field in which p remains inert and $\Psi : \mathrm{Gal}\,(\bar{\mathbb{Q}}/K) \longrightarrow \mathbb{C}^\times$ be a dihedral character of K of conductor relatively prime to $p \cdot \mathrm{disc}(K)$.

It is not difficult to verify that the order of vanishing of the L-function $L(E/K, \Psi, s)$ associated to the twist of E/K by Ψ at $s = 1$ is odd. One actually expects this order of vanishing to be equal to 1 in almost all cases. In this scenario, BSD predicts the existence of a non-trivial *global* point in the Ψ-isotypical component

$$E(H)[\Psi] = \{P \in E(H) \otimes \mathbb{C} \quad \text{such that} \quad \sigma(P) = \Psi(\sigma)P \quad \text{for all } \sigma \in \mathrm{Gal}(H/K)\}$$

of the Mordell-Weil group of E over the abelian extension H/K cut out by Ψ. As soon as Ψ is not quadratic, the theory of Heegner points becomes helpless and one is at a loss for constructing the putative point that is expected to exist. Let K_p denote the p-adic completion of K, a quadratic unramified extension of the field $\mathbb{Q}_p$ of p-adic numbers. In 1999, H. Darmon invented his theory of Stark-Heegner points (now often called Darmon points), supplying a collection

$$\{z_\Psi\} \subset E(K_p) \otimes \mathbb{C}$$

of points of pure local nature, indexed by dihedral characters of K as above.

Darmon formulated a revolutionary conjecture on their global rationality, whose proof would give rise to the sought-after points in $E(H)[\Psi]$ as predicted by BSD. We refer the reader to his address [D] at the International Congress of Mathematicians celebrated in Madrid in 2006.

However, until recently Darmon's conjecture remained completely open. The main contribution of my recent works [DR3], [DR4] in collaboration with Henri Darmon is proving this conjecture up to the finiteness of the relevant Tate-Shafarevic group, under minor technical hypotheses.

In order to state our result more precisely, let $\mathrm{Sel}_p(E/H)[\Psi]$ denote the Ψ-isotypic component of the global Selmer group of the base change of E to the class field H. This group is a sort of cohomological stand-in for $E(H)[\Psi]$, the Ψ-isotypic component of the Mordell-Weil group of E over H, which is where Darmon's Stark-Heegner points z_Ψ are conjectured to lie. More precisely, $\mathrm{Sel}_p(E/H)[\Psi]$ contains $E(H)[\Psi]$, and the extent to which this inclusion fails to be an equality is measured by a pro-p Tate-Shafarevic group. Since the latter is conjectured to be trivial, one expects all global Selmer classes should actually belong to $E(H)[\Psi]$.

The main theorem we prove in [DR3], [DR4] may be roughly stated as follows in the above scenario, up to an analytic non-vanishing assumption that is always expected to hold.

Theorem 2 (Darmon-Rotger)**.** *There exists a global Selmer class κ_Ψ in $\mathrm{Sel}_p(E/H)[\Psi]$ such that its local component at p is Darmon's Stark-Heegner point z_Ψ.*

Idea of proof

The main idea underlying the proof of the main theorem in [DR3], [DR4] consists in recasting the statements we aim to prove in terms of a p-adic L-function associated to a triple (f, g, h) of p-adic *families* of ordinary overconvergent modular forms, which for simplicity may be denoted $L_p(f, g, h)$. This notation is however oversimplifying, as it sweeps under the rug some of the most delicate properties of this p-adic L-function. Indeed, this three-variable function is constructed by interpolating the square-root of the central critical value of the complex Garrett L-function associated to a triple $f(k)$, $g(\ell)$, $h(m)$ of classical specializations of the families f, g and h, divided by a suitable period.

The weights k, ℓ and m of these specializations must be chosen in such a way that the sign of the functional equation of the motive associated to the triple $(f(k), g(l), h(m))$ is $+1$. Under minor technical assumptions, this amounts to saying that one of the weights is larger than or equal to the sum of the other two.

This automatically gives rise to three different natural regions of interpolation, depending on which of the three weights is taken to be the dominant one. In consequence there arise three different p-adic L-functions, depending on the chosen region of interpolation, that we call $L_p(f; g, h)$, $L_p(g; f, h)$ and $L_p(h; f, g)$, respectively. The family that appears first corresponds to the one whose weight is dominant in the region of interpolation, and the period intervening in the interpolation formula only depends on this family. The order in which the latter two families appear is irrelevant.

Choose the above families in such a way that f specializes in weight two to the eigenform associated by Wiles to the elliptic curve E, and let g and h be cuspidal Hida families specializing in weight one to eigenforms whose associated Artin representations satisfy that their tensor product contains as an irreducible constituent the two-dimensional induced representation $\mathrm{Ind}(\Psi)$ from K to $\mathbb{Q}$.

Thanks to the work of Bertolini, Seveso and Venerucci [BSV], one can show that the second partial derivative of $L_p(f; g, h)$ along the first family at the triple of weights $(2, 1, 1)$ is a multiple of the logarithm of a global class κ in the Selmer group of the twist of E/K by Ψ. As explained in [DR3], our theorem follows after showing that $L_p(f; g, h)$ factors as the product $L_p(f; g, h) = L_p(E/K, \Psi, s) \cdot L_p(E/K, \chi, s)$ of two further p-adic L-functions, up to auxiliary terms; the first derivative of the first factor at the point $(2, 1, 1)$ encodes Darmon's Stark-Heegner point, while the first derivative of the second factor is a non-vanishing period Ω_χ thanks to our running assumption. The sought-after class κ_Ψ is obtained by taking $\kappa_\Psi = \frac{1}{\Omega_\chi} \cdot \kappa$.

A non-trivial problem one encounters at the outset is the fact that the parameter space of the Euler system that Darmon and myself constructed long ago in [DR2] was only one-dimensional, while the proof of the conjecture required a much more flexible Euler system, varying freely along the three-dimensional weight space afforded by the three weights of the Hida families f, g and h. This was addressed in [DR4]. ▷ Editorial

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Bell correlations in quantum many-body systems

by Jordi Tura (Instituut-Lorentz, Leiden).
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Quantum mechanics allows for correlations that cannot be explained within two principles that are fundamental to our intuitive understanding of the world: locality and realism [1]. This led to Einstein and collaborators to wonder whether quantum mechanics was a complete theory for the description of reality [2]. Albeit these discussions remained pretty much on the philosophical plane, John S. Bell eventually formalized them via the introduction of a local hidden variable model (LHVM) [3]. Quantum correlations that do not admit a LHVM are termed *nonlocal*. Nonlocal correlations are revealed through the violation of a so-called *Bell inequality*. Incidentally, this year's Nobel Prize in Physics rewarded the efforts of Aspect [4], Clauser[5] and Zeilinger [6, 7] *for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science*.

An LHVM is defined through an experiment where n space-like separated parties perform measurements on their laboratories and record their outcomes. Their measurement choices should be statistically independent of the internal state of the system being measured. Space-like separation prevents communication of the measurement choices to each other. We then expect to observe correlations of the form

$$P(\mathbf{a}|\mathbf{x}) = \int_\Lambda p(\lambda)\prod_{i=1}^{n} p_i(a_i|x_i\lambda)\mathrm{d}\lambda, \tag{1}$$

where $P(\mathbf{a}|\mathbf{x})$ denotes the probability that the i-th party observed outcome a_i given that they performed the x_i-th measurement. Since parties are not allowed to communicate during the experiment, but may have done so in the past (an interaction which is encoded in the so-called hidden variable $\lambda \in \Lambda$), we expect their individual responses to be independent if conditioned on λ. Although we do not even define in which space λ may live, this is integrated out in Eq. (1).

Geometrically speaking, LHVM correlations form a convex polytope, denoted by $\mathbb{P}$. Its facets correspond to tight Bell inequalities and its vertices v_i are enumerated by all local deterministic strategies (LDSs); i.e., when each $p_i(a_i|x_i,\lambda)$ is a Kronecker delta [8]. The vertices are straightforward to generate, but finding a minimal complete set of Bell inequalities is extremely costly [9] requiring use of the dual description method, which scales as $O(V^{\lfloor D/2 \rfloor})$, where $V = |\{v_i\}_i|$ is the number of vertices and D the ambient space dimension in which $\mathbb{P}$ is embedded [10]. In a Bell experiment with n parties, m measurement choices per party and d possible outcomes per measurement, $V = d^{nm}$ and $D = (1+m(d-1))^n - 1$, which makes the task of obtaining all Bell inequalities intractable except for the simplest scenarios [11].

One might try to balance complexity and expressivity by focusing on Bell inequalities of a particular form [12, 13], further adapting them to overcome experimental limitations [14]. A natural choice is to look for Bell inequalities invariant under the action of a symmetry group G; e.g., permutationally invariant Bell inequalities (PIBIs), where $G = \mathfrak{S}_n$ [15]. That requires characterizing the projected polytope $\mathbb{P}^G := \pi(\mathbb{P})$ onto the G-invariant subspace, where

$$\pi : P(\mathbf{a}|\mathbf{x}) \mapsto \frac{1}{|G|}\sum_{\sigma\in G\leq\mathfrak{S}_n} P(\sigma(\mathbf{a})|\sigma(\mathbf{x})). \tag{2}$$

The projection π naturally induces a partition of the multipartite LDSs into disjoint orbits, through the action of the symmetry group G. Via Pólya's enumeration theorem, there are $|\{\pi(v_i)\}_i| = |G|^{-1}\sum_{\sigma\in G} d^{c(\sigma)}$ classes, where c counts the number of disjoint cycles of σ. The latter is an upper bound on the number of projected vertices, since π needs not preserve extremality [16].

In the case of $G = \mathfrak{S}_n$, the orbits are enumerated by partitions of n into d^m elements. For convenienece, let us denote $\mathbf{c} \vdash n$ such a partition, where $c_\mathbf{i}$ (with $\mathbf{i} = i_1 \ldots i_m$ encoded in base d) denotes how many parties choose the strategy that maps the j-th measurement to the i_j-th outcome [15]. At every LDS correlations behave deterministically, forming Boole algebra [17]. At the level of Eq. (1), this means that $P(\mathbf{a}|\mathbf{x}) = \prod_i p_i(a_i|x_i)$. This factorization is also reflected at the level of $\mathbb{P}^{\mathfrak{S}_n}$, where the $\mathfrak{S}_n$-symmetric k-body marginals are k-degree polynomials in $\mathbb{R}[\mathbf{c}]$ [15, 18].

Although the computational complexity of $\mathbb{P}^{\mathfrak{S}_n}$ is greatly reduced, the number of vertices is now upper bounded by $\binom{n+d^m-1}{d^m-1}$ and if we further restrict $\mathbb{P}^{\mathfrak{S}_n}$ to the space of at most k-body correlators, denoted $\mathbb{P}_k^{\mathfrak{S}_n}$, then $D = O(\text{poly}(k))$, the doubly-exponential scaling of the double-description method still prohibits a complete description for moderate values of n in practice.

Since $\mathbb{P}_k^{\mathfrak{S}_n}$ is the convex hull of the points whose coordinates are expressed via elements of $\mathbb{R}[\mathbf{c}]$, when evaluated at partitions of n, one may further relax this condition to gain in terms of computational efficiency, at the expense of losing a bit in terms of resolution. In this case, the condition $c_\mathbf{i} \geq 0, \sum_\mathbf{i} c_\mathbf{i} = n, c_\mathbf{i} \in \mathbb{Z}$ can be relaxed to $c_\mathbf{i} \in \mathbb{R}$. This transforms the problem of finding the convex hull of a finite number of points to that of finding the convex hull of a semialgebraic set that interpolates through the original one when $\mathbf{c} \vdash n$. That semialgebraic set is defined through the ideal resulting from the polynomial equations satisfied at LDSs, and the equality and inequality constraints inherited from $\mathbf{c} \vdash n$ [19].

The membership problem in the convex hull of a semialgebraic set is NP-hard in general, but approximations exist through semidefinite programs (SDP) [20, 21]. In particular, via the so-called moment problem formulation [22, 23], the following SDP

$$\begin{array}{llll} \max_{\mathbf{y}\in\mathbb{R}^M} & 0 & & \\ \text{s.t.} & \sum_i y_i \Gamma_i & \succeq & 0 \\ & y_0 & = & 1 \\ & y_j & = & (\mathcal{S}^*)_j \quad 1 \leq j \leq K, \end{array} \tag{3}$$

where $K < M$ and Γ_i are symmetric matrices encoding the semialgebraic set relations, can quickly certify (in $O(1)$ complexity with respect to n) if the experimental correlations, encoded in the vector $\mathcal{S}^*$, lie outside the convex hull of the semialgebraic set [19]: if the SDP (3) is infeasible, it yields a certificate through its dual formulation. Such a certificate is the Bell inequality that the experimental data $\mathcal{S}^*$ violates. The Bell inequality coefficients and classical bound correspond to the dual variables associated to the equality constraints in (3). This method has successfully bypassed the polytope approach already in mesoscopic systems ($500 \lesssim n \lesssim 5 \cdot 10^5$) [14, 24]. A natural next step is its application to find Tsirelson's bounds to multipartite inequalities (what is the maximal quantum violation that Nature allows for a Bell inequality), where the polynomials are non-commutative [25], yielding operator-sum-of-squares decompositions applicable to self-testing [26]. Under certain conditions, the latter allows to certify, just from the observed statistics, which quantum states and measurements (up to unobservable degrees of freedom) must have generated them. ▷ Editorial

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PhD highlights

Mar Giralt Miron defended her PhD thesis *Homoclinic and chaotic phenomena to L_3 in the Restricted 3-Body Problem* on November 25th, 2022. The thesis was produced within the UPC doctoral program in Applied Mathematics and was supervised by Inma Baldomá Barraca and Marcel Guàrdia Munárriz. Currently she is a postdoc at the Università degli Studi di Milano.

Thesis summary: The Restricted 3-Body Problem models the motion of a body of negligible mass under the gravitational influence of two massive bodies, called the primaries. If the primaries perform circular motions and the massless body is coplanar with them, one has the Restricted Planar Circular 3-Body Problem (RPC3BP). In synodic coordinates, it is a two degrees of freedom autonomous Hamiltonian system with five critical points, $L_1, \ldots, L_5$, called the Lagrange points. The Lagrange point L_3 is a saddle-center critical point which is collinear with the primaries and is located beyond the largest one.

In this thesis we study some of the homoclinic and chaotic phenomena occurring around L_3 and its stable and unstable manifolds when the ratio between the masses of the primaries μ is small.

In Part I and II of the thesis, we obtain an asymptotic formula for the distance between the unstable and stable manifolds of L_3, see [1, 2]. When μ is small the eigenvalues associated with L_3 have different scales, with the modulus of the hyperbolic eigenvalues smaller than the elliptic ones. Due to this rapidly rotating dynamics, the invariant manifolds of L_3 are exponentially close to each other with respect to $\sqrt{\mu}$ and, therefore, the classical perturbative techniques (i.e. the Poincaré-Melnikov method) cannot be applied. Then, one infers that there do not exist 1-round homoclinic orbits, i.e. homoclinic connections that approach the critical point only once.

In Part III of the thesis, we apply the asymptotic formula obtained in the previous parts to prove the existence of homoclinic and chaotic phenomena around L_3 and its invariant manifolds, see [3]. Firstly, we study the existence of 2-round homoclinic orbits to L_3, i.e. connections that approach the critical point twice. More concretely, we prove that there exist 2-round connections for a specific sequence of values of the mass ratio parameters. We also obtain an asymptotic expression for this sequence.

Moreover, we prove the existence of Lyapunov periodic orbits whose two dimensional unstable and stable manifolds intersect transversally. The family of Lyapunov periodic orbits of L_3 has Hamiltonian energy level exponentially close to that of the critical point L_3. Then, by the Smale-Birkhoff homoclinic theorem, this implies the existence of chaotic motions (Smale horseshoe) in a neighborhood exponentially close to L_3 and its invariant manifolds.

In addition, we also prove the existence of a generic unfolding of a quadratic homoclinic tangency between the unstable and stable manifolds of a specific Lyapunov periodic orbit, also with Hamiltonian energy level exponentially close to that of L_3.
▷ Editorial

Highlighted publication: [1]

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Tuomas Hakoniemi defended his PhD thesis *Size bounds for algebraic and semialgebraic proof systems* on March 03, 2022. The thesis was produced within the UPC doctoral program on Computing and his advisor was Albert Atserias. Currently, he is a postdoctoral researcher at Imperial College London in the Iddo Tzameret group.

Thesis summary: Proof complexity is a field in the crossroads of mathematical logic and computational complexity theory that studies the resources needed to prove formal statements in different logical calculi. In 1979, Cook and Reckhow proposed a general definition of a propositional proof system as a poly-time computable function onto the set of tautologies. With this definition at hand they made the simple but foundational observation that there is no propositional proof system for classical propositional logic that has short, i.e. polynomial-sized, proofs for all tautologies unless NP is closed under complementation.

Besides logic, different proof calculi can be found also in other parts of mathematics. In this thesis we study calculi arising from algebraic geometry and combinatorial optimization. We consider the proof complexity of algebraic and semialgebraic proof systems Polynomial Calculus, Sums-of-Squares and Sherali-Adams. These systems are refutation systems for unsatisfiable sets of polynomial constraints. Polynomial Calculus, introduced by Clegg, Edmonds and Impagliazzo, has its roots in the theory of Gröbner bases in computational algebraic geometry, while Sums-of-Squares and Sherali-Adams proof systems stem from the hierarchies of semidefinite and linear relaxations in polynomial optimization introduced by Lasserre, and Sherali and Adams, respectively.

The most studied complexity measure for these systems is the degree of the proofs, not least due to the close connections to the levels in the hierarchies mentioned above. This thesis concentrates, however, on other possible complexity measures of interest to proof complexity, monomial-size and bit-complexity, i.e. the number of (distinct) monomials needed in the proofs and the number of bits needed to represent the proofs (over rationals) in binary notation. We aim to showcase that there is a reasonably well-behaved theory for these measures also.

Firstly we tie the complexity measures of degree and monomial-size together by proving size-degree trade-offs for

Sums-of-Squares and Sherali-Adams. In more detail, we show that if there is a refutation with at most s many monomials, then there is a refutation of degree $O(\sqrt{n \log s} + k)$, where k is the maximum degree of the constraints and n is the number of variables. This gives us a criterion for monomial-size lower bounds from degree lower bounds. These results are analogous to the celebrated size-width trade-off for Resolution of Ben-Sasson and Wigderson. For Polynomial Calculus similar trade-off was obtained earlier by Impagliazzo, Pudlák and Sgall.

Secondly we prove a form of feasible interpolation for all three systems. Feasible interpolation is a framework introduced by Krajíček for reducing lower bounds in proof complexity to lower bounds in other computational models. The basic form of feasible interpolation takes a refutation of two formulas $\varphi(x, z)$ and $\psi(y, z)$ with disjoint sequences x, y and z of variables, and extracts from it a somehow feasible algorithm computing an interpolant of the formulas, i.e. an algorithm that given an assignment a to the z-variables decides whether $\varphi(x, a)$ or $\psi(y, a)$ is unsatisfiable. We show that for each of the three systems there is a polynomial time algorithm that given a refutation of two mutually unsatisfiable sets $P(x, z)$ and $Q(y, z)$ of polynomial constraints and an assignment a finds a refutation of either $P(x, a)$ or $Q(y, a)$.

Finally we consider the relationship between monomial-size and bit-complexity in Polynomial Calculus and Sums-of-Squares. We exhibit an unsatisfiable set of polynomial constraints that has both Polynomial Calculus and Sums-of-Squares refutations with only polynomially many distinct monomials, but for which any Polynomial Calculus or Sums-of-Squares refutation requires exponential bit-complexity.

Besides the emphasis on complexity measures other than degree, another unifying theme in all the three results is the use of semantic characterizations of resource-bounded proofs and refutations. All results make heavy use of the completeness properties of such characterizations: we argue for the existence of resource-bounded proofs and refutations from the non-existence of suitable semantic objects, and for the existence of the semantic objects from the non-existence of resource-bounded proofs or refutations. All in all, the work on these semantic characterizations presents itself as the fourth central contribution of this thesis. ▷ Editorial

Highlighted publication:

Albert Atserias and Tuomas Hakoniemi. Size-Degree Trade-Offs for Sums-of-Squares and Positivstellensatz Proofs. In Proceedings of the 34th Computational Complexity Conference (CCC 2019), *Leibniz International Proceedings in Informatics* (LIPIcs) **137** (2019), pages 24:1–24:20. Published by Schloss Dagstuhl–Leibniz-Zentrum für Informatik, edited by Amir Shpilka.

Guillem Belda-Ferrín defended his PhD thesis *Conformal n-dimensional Bisection for Local Refinement of Unstructured Simplicial Meshes* on October 28th, 2022. The thesis was produced within the UPC doctoral program in Applied Mathematics and was supervised by Xevi Roca and Eloi Ruiz-Gironés. Currently, he is working as algorithm developer at Hexagon Manufacturing Intelligence designing path planning algorithms.

Thesis summary. In n-dimensional adaptive applications, conformal simplicial meshes must be locally modified. One systematic local modification is to bisect the prescribed simplices while surrounding simplices are bisected to ensure conformity. Although there are many conformal bisection strategies, practitioners prefer the method known as the newest vertex bisection [7–10]. This method guarantees key advantages for adaptivity whenever the mesh has a structure called reflectivity. Unfortunately, it is not known (i) how to extract a reflection structure from any unstructured conformal mesh for three or more dimensions. Fortunately, a conformal bisection method is suitable for adaptivity if it almost fulfills the newest vertex bisection advantages. These advantages are almost met by an existent multi-stage strategy in three dimensions [11]. However, it is not known (ii) how to perform multi-stage bisection for more than three dimensions.

This thesis aims to demonstrate that n-dimensional conformal bisection is possible for local refinement of unstructured conformal meshes. To this end, it proposes the following contributions. First, it proposes the first 4-dimensional two-stage method [6], showing that multi-stage bisection is possible beyond three dimensions. Second, following this possibility, the thesis proposes the first n-dimensional multi-stage method [1], and thus, it answers question (ii). Third, it guarantees the first 3-dimensional method that features the newest vertex bisection advantages [5], showing that these advantages are possible beyond two dimensions. Fourth, extending this possibility, the thesis guarantees the first n-dimensional marking method [3] that extracts a reflection structure from any unstructured conformal mesh, and thus, it answers question (i). This answer proves that local refinement with the newest vertex bisection is possible in any dimension. Fifth, this thesis shows that the proposed multi-stage method almost fulfills the advantages of the newest vertex bisection [2]. Finally, to visualize four-dimensional meshes, it proposes a simple tool to slice pentatopic meshes [4].

In conclusion, this thesis demonstrates that conformal bisection is possible for local refinement in two or more dimensions. To this end, it proposes two novel methods for unstructured conformal meshes, methods that will enable adaptive applications on n-dimensional complex geometry. ▷ Editorial

Highlighted publication: [1].

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Outreach

Why and how do cells migrate? by Pablo Sáez (LaCàN, **IMTech**)

Received on November 27th, 2022

Cell migration is a mechanical function central to life. It establishes how cells move and interact during development to eventually form fully functional organs. When this fascinating and yet obscure interaction of genetics, chemistry and physics evolves normally, our brain, heart, liver, etc., work in perfect cooperation. Consequently the body performs without complains. A fascinating property of organ development is that it is extremely robust. If we consider the possibilities of the number of alterations in human development that could occur due to abnormal cell and tissue function, and that eventually most of the organs develop without an issue, we will appreciate how exceptional our body is. The equally robust cell migration is not only the very essence of a successful organ development but it is also important for regenerative properties of damaged tissues and organs.

Unfortunately, the robustness of biological systems has also important drawbacks. For example, cell migration is responsible for the invasion of cancer cells from of the tumor niche toward healthy tissues, through the blood or lymph system. Similar to the development and regeneration of tissues, tumor cell invasion is a very robust process. When this happens, the tumor has metastasized. This is a dramatic stage in cancer progression, because when tumor cells leave the primal tumor and advance to other tissues and organs, cancer is hard to control.

Traditionally, experimental work has taken the reins in the study of cell, tissue and organ biology. This was also the case for cell migration. However, mathematicians, physicists and engineers have been very active during the last decades to build accurate mathematical models that explain experimental observations and provide new insights on how cells migrate. Most mathematical models of cell migration have relied on three key cellular structures associated with basic mechanical processes (see Figure). Cells are made of a dynamic cytoskeleton network encapsulated in the cell membrane. One of these networks, the actin network, is directly involved in cell migration. The actin network continuously polymerizes and pushes the cell membrane outward. This is the first structure associated with cell migration. A second distinctive actin network combines with myosin motors, molecules that link two actin filaments and pull on them, to create an active viscoelastic. This acto-myosin network, also named retrograde flow, flows inward from the cell boundary. The result of these two competing actin networks, one moving outward and the other inwards, determine the direction and velocity of cell migration. Cells also create cell adhesions, clusters of molecules that link the intracellular acto-myosin network with the extracellular matrix (ECM), to sense their mechanical environment. Moreover, cell adhesion dynamics depends on how these molecules sense and respond to the mechanical properties of the ECM and determines how cell adhesions form and disassemble. In physical terms, cell adhesion creates friction between the cell and the ECM.

But then, how does a cell determine the direction of migration? How does a cell organize itself to migrate in a specific direction during development, regeneration or cancel invasion? Without a stimulus that can break the symmetry in all the mechanical mechanisms described above, cells would spread symmetrically but they would not migrate. Cells follow signals of very different nature that are self-generated or exogenous.

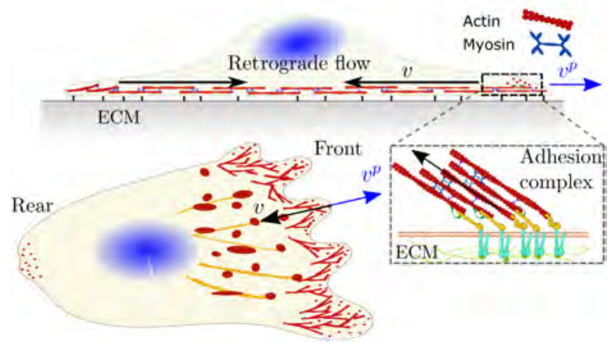

Figure. Sketch of the cell system. Main mechanisms acting in cell migration: cell adhesion, retrograde flow and polymerization of actin against the cell membrane. v^p (blue arrows) and v (black arrows) are the polymerization and retrograde flow velocities.

For example, cells move guided by chemical and mechanical signals, in-vivo and in-vitro. In an environment where chemical gradients exist, different types of molecules that can cross the cell membrane feel the slightest of chemical gradients that exist around the cell body. Then, the cell activates intracellular chemical signals and polarize following the external chemical gradient. These chemical signals modify the activity of actin polymerization or myosin activity and, therefore, leads to a cell polarization in terms of the acto-myosin activity. Eventually, the acto-myosin polarization determines the directed cell migration during chemotaxis. Similarly, when the ECM in which a cell is embedded presents gradients of its mechanical properties, the cell senses it through the cell adhesion molecules. If one side of a cell is located in a stiff region but other side is on a softer one, the formation and strength of the cell adhesions, and consequently cell-ECM friction, would differ at different regions of the cell. As a result, the intracellular flow on the stiffer region will be different than the one on the softer region and this asymmetry would make the cell to undergo directed cell migration.

In the LaCàN, we have developed a number of mathematical models to study cell migration. Some of the simplest models, but still comprehensive enough to study cell migration, are those based on active gel theory [1]. We can also simplify the model assuming a 1D domain Ω moving in time with coordinates $x(t) \in [l_r(t), l_f(t)]$. $l_r(t)$ and $l_f(t)$ represent the rear and front boundary of the cell. These class of models determine the actin flow velocity v by solving balance of linear momentum in the following form:

$$\partial_x(\mu \partial_x v + \zeta \rho) = \eta v \quad \text{in} \quad \Omega. \tag{4}$$

This balance assumes a viscous behavior of the actomyosin network, $\mu \partial_x v$, introduces the active contractility of myosin motors, ζ, and the cell-ECM friction, η, generated by the formation of cell adhesions. The acto-myosin network that forms at the cell-ECM contact plane usually flows inward, from the cell periphery toward the cell nucleus. The actin network that polymerizes against the cell membrane has a maximum polimerization velocity v_0^p when the cell membrane is under no tension. When the cell membrane builds up in tension, which occurs during the first stages of cell migration, the polymerization velocity, v^p, decreases proportionally to the opposing tension of the membrane point-wise. The outward polymerization velocity, v^p, competes with the inward retrograde flow velocity, v, to expand or retract the cell periphery (Figure). Active gel models also introduce mass transport of the intracellular. We can quantify the network density $\rho(x,t)$ as

$$\partial_t \rho + \partial_x f(\rho) = k_p - k_d \rho \quad \text{in} \quad \Omega, t > 0, \tag{5}$$

where $f(\rho) = (w\rho - \nu \partial_x \rho)$. The network velocity drags the cytoskeleton with velocity w in the cell frame of reference, diffuses and turnover with polymerization and depolymerization rates k_p and k_d. Different types and values of boundary conditions can be imposed to Eq. 4 and 5 to model specific phenomena. By coupling these basic equations, we can quantify what are the precise mechanical forces required for the cell to migrate and thus analyze how exogenous stimuli may induce a directed cell migration.

Recently, we have studied durotaxis, the directed cell migration toward stiffness gradients, through a mathematical model. A key aspect of our research was to couple stochastic clutch models, which describe the dynamic behavior of cell adhesions) [2] into the active gel theory. This model allowed us to explain how certain cell types migrate toward positive gradients of the ECM stiffness while others do it toward negative gradients. With an extension of this previous model of durotaxis, we have also studied how cells sense and react to chemical stimuli to, eventually, migrate toward chemical gradients. More importantly, we analyzed how these two chemical and mechanical stimuli compete with each other [3]. This is a very important aspect in cell migration because, in in-vivo and in-vitro, cells are usually in complex mechano-chemical context, where both signals, at a smaller or larger degree, coexist.

We have also used these models to address how tumor cells may escape the tumor mass and invade other tissues, within the Mechano-control Project (FET proactive). During cancer progression, the ECM experiences mechanical changes, which are induced by the cell themselves. As a consequence, the cell adhesion changes, and so the cell-ECM friction does [4]. These adhesion changes induce remodeling of the intracellular network which trigger an oncogenic response. These changes in the ECM mechanics and in cell adhesion are critical in the invasiveness of tumor cells. Essentially, this change resembles the mechanism of durotaxis. Clearly, the impact of understanding the mechanical interactions between the cell and the ECM works is huge, because, if we understand how tumor cells migrate, we could design strategies for arresting tumor cell invasion.

Nevertheless, the implications of understanding how cells migrate goes further than in life sciences. It also has an important impact in engineering. Throughout its history, engineering has generally used manufactured materials to build solutions such as airplanes, robots or electronic devices. While many of these engineering designs were somewhat bio-inspired, they are still far from achieving some of the unique functionalities of living organisms. Think, for example, on the robustness of our heart or the maneuverability of an eagle compare to jet aircrafts. Recent technologies have used assemblies of artificial materials and biological tissues to recapitulate more closely certain behaviors of biological systems. This technology has been used for the design of bio-hybrid soft-robots, that, e.g., imitate the movement of octopus arms, and for the design of grafts to support cardiac function. If we learn how to manipulate the directed cell migration, it may be the first step to engineer novel tissue constructs that form autonomously and that, upon formation, closely resembles the biological system they were intended to imitate. ▷ Editorial

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Chronicles

The first five items of his section are devoted to the ICM-2022 events related to the lectures and work of the Fields Medalists (Jaymes Maynard, June Huh, Hugo Duminil-Copin, Maryna Viazovska), and the Chern Medal (Barry Mazur).

James Maynard: a Fields Medal for major advances in Analytic Number Theory

by Juanjo Rué (**IMTech**, DMAT)

Received on September 16th, 2022

As it is stated in Maynard's laudatio, this Fields Medal has been awarded *for contributions to Analytic Number Theory, which have led to major advances in the understanding of the structure of prime numbers, and in Diophantine approximation*. We will describe three of his main contributions.

The first two deal with prime numbers. Maynard's most celebrated result is his proof on short gaps between consecutive prime numbers. If we write $\mathbb{P} = \{p_1 < p_2 < \dots\}$ for the set of prime numbers, what is $s = \liminf\{p_{k+1} - p_k\}$? In 2005, Goldston, Pintz and Yildirim [5] showed that

$$\liminf \frac{p_{k+1} - p_k}{\log(p_k)} = 0,$$

which shows that small gaps are at most logarithmic. In May 2013 Zhang [11] made a major advance by beating the logarithmic barrier and showing that $\liminf\{p_{k+1} - p_k\} < 7 \times 10^7$. After this breakthrough, a Polymath project (lead by Terry Tao) was initiated in order to optimize Zhang's bound. In November 2013 Maynard [7] came up with a new bright idea (by improving the sieve methods developed by Goldston, Pintz and Yildirim) and reducing the upper bound to 600. Combining all ideas from Polymath and the new analytic techniques from Maynard it was possible to reduce the bound until 246 (also, assuming the so-called generalized Elliott-Halberstam conjecture [2], this bound can be reduced to 6; see the Polymath webpage for a history of the updates on the constant).

Maynard's second main contribution deals with the opposite problem, namely large gaps between consecutive primes. The question now is: which is the growth rate of $L(n) = \max_{p_{k+1}\leq n}\{p_{k+1} - p_k\}$? Due to a work of Pintz [9], the best result known was that

$$L(n) \gg c(n)\frac{\log(n)\log\log(n)\log\log\log\log(n)}{(\log\log\log(n))^2}, \quad c(n) < 2e^{\gamma},$$

where γ stands for the Euler's constant. Erdös offered a $10.000 prize to show that $c(n)$ can be taken arbitrarily large. This result was proved to be correct in 2014 by Ford-Green-Konyagin-Tao [4] and, independently and using different techniques, by Maynard [8]. By combining both techniques and using some new covering results arising from extremal combinatorics, the union of both teams were able to push the previous growth barrier to

$$L(n) \gg c\frac{\log(n)\log\log(n)\log\log\log\log(n)}{\log\log\log(n)},$$

for infinitely many values of n, [3]. Those advances solved the higher prize Erdös ever offered for a proof of a mathematical question.

The third main Maynard's contribution is in the area of Diophantine approximation. This subarea of Number Theory deals with the following question: which real numbers have a "good" approximation in terms of rational numbers? The very first result in this area is Dirichlet theorem (which can be proved by using the pigeonhole principle), which states that for every irrational real number α there exists infinite pairs of integers (p_n, q_n) such that

$$\left|\alpha - \frac{p_n}{q_n}\right| < \frac{1}{q_n^2}.$$

Dirichlet's Theorem was widely strengthen by Roth by proving the celebrated Thue-Roth-Siegel theorem (see [10]), which states that if α is now an *algebraic* irrational integer, then for all $\varepsilon > 0$ there exists only a finite number of choices for integers (p_n, q_n) such that

$$\left|\alpha - \frac{p_n}{q_n}\right| < \frac{1}{q_n^{2+\varepsilon}}.$$

Roth won a Fields Medal on 1958 for this result (as well as for his solution of the quantitative version of Van der Vaerden Theorem for 3-arithmetic progressions).

A proposal to strength Dirichlet theorem was given by Duffin and Schaeffer in 1941 [1] as follows: instead of considering the error q_n^{-2}, consider $\frac{f(q_n)}{q_n}$, where f is a given function. His conjecture was the following (named as Duffin-Schaeffer conjecture): given $f:\mathbb{N}\to\mathbb{R}_{\geq 0}$, the following two statements are equivalent:

(1) $\sum_{n\geq 1} f(n)\frac{\varphi(n)}{n}$ diverges (where $\varphi(n)$ denotes the Euler totient function).

(2) For almost all $\alpha\in\mathbb{R}$, the inequality $\left|\alpha - \frac{p}{q}\right| < \frac{f(q)}{q}$ has infinitely many solutions for p and q coprime numbers.

Observe that taking $f(n) = 1/n$ we rediscover Dirichlet theorem, and also that $\sum_{n\geq 1}\frac{\varphi(n)}{n^2}$ is divergent. Duffin and Schaeffer already proved that (2) implies (1) (the argument is based on a simple application of Borel-Cantelli lemma in Measure Theory) but the converse was an open conjecture until Maynard, joint with Koukoulopoulos proved this result in 2019, [6].

Those three examples are only a small description of the important contributions of Maynard in the understanding of number theory and open new horizons in a very classical area in pure mathematics. ▷ Editorial

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Maryna Viazovska by Joaquim Ortega Cerdà (Department of Mathematics and Computer Science, UB). *Received on 21 November, 2022.*

The most spectacular result by Marina Viazovska is the description of the optimal packing by balls in dimensions eight and twenty-four, [3, 6]. I will concentrate on presenting this result, although she has many other interesting contributions. This problem consists in finding the first order asymptotic of the maximum number of disjoint unit balls that one can fit in a cube of radius R as R becomes very big. It has a trivial solution in dimension one. In dimension two, the hexagonal lattice configuration is optimal and in dimension three, the problem is known as the Kepler conjecture, and it was solved by Hales, [4]. The optimal configuration in this case is provided by the density of the cannonball stack.

This problem has a reformulation in potential theory: To find the minimum of the potential energy among all possible configurations of points in $\mathbb{R}^d$ with a given density ρ. The density of a sequence of points $\mathcal{A}\subset\mathbb{R}^d$ is given by

$$\rho = \lim_{R\to\infty}\frac{\#(B(0,R)\cap\mathcal{A})}{|B(0,R)|}.$$

Given a sequence of points $\mathcal{A}$ with density ρ, its energy with respect to a given potential function p is defined as

$$E_p(\mathcal{A}) = \liminf_{R\to\infty}\frac{\sum_{x\neq y\in\mathcal{A}\cap B(0,R)} p(|x-y|)}{\#(B(0,R)\cap\mathcal{A})}.$$

It is also of interest to know the geometric distribution of the optimal configurations $\mathcal{A}$. Of course, the problem depends on the dimension d of the ambiance space and a priori on the potential p. It happens, though, that for a large family of potentials, at least in some dimensions, the optimal configuration is always the same. If this is the case, we say that the optimal configuration $\mathcal{A}$ is universally optimal.

The spherical packing problem can be seen as a degenerate case of this problem when $p(x) = \chi_{(0,2)}(r)$. In this case, a collection of points $\mathcal{A}$ is a packing by spheres if $E_p(\mathcal{A}) = 0$. Thus, to find the optimal spherical packing is equivalent to find a sequence $\mathcal{A}$ with the biggest density ρ possible, so that A has zero energy, $E_p(\mathcal{A}) = 0$.

The approach to solve this problem in dimension eight and twenty-four is the so-called linear programming bound technique introduced by Cohn and Elkies in [1]. In these two dimensions there were candidates to be the best configuration of points (the E_8 and the Leech lattice), so the problem was reduced to prove that they were optimal. The theorem of Cohn and Elkies postulates that if there is a function f in the Schwartz class such that $f(x) \leq p(|x|)$ and $\hat{f}(\omega) \geq 0$, then $E_p(\mathcal{A}) \geq \rho\hat{f}(0) - f(0)$, for all configurations $\mathcal{A}$ of density ρ. This result is a consequence of the classical Poisson summation formula.

The difficulty, then, lies in finding the "magical function" f that provides the optimal density bound. This is what Viazovska did in [6] for the potential p associated to the spherical packing problem. And in a recent work, [2], they extended this result to prove that E_8 and the Leech lattice are universally optimal for all potentials that are completely monotonic in the square of the distance.

The tools to build this optimal function f are a Fourier interpolation formula, and the key building functions require a delicate study of modular forms. The Fourier interpolation formula is surprisingly new and of independent interest, and it was more systematically explored in [5], where Radchenko and Viazovska proved that any even function f of the Schwartz class in $\mathbb{R}$ has the representation:

$$f(x) = \sum_{n=0}^{\infty} f(\sqrt{n}) a_n(x) + \sum_{n=1}^{\infty} \hat{f}(\sqrt{n}) \hat{a}_n(x).$$

where a_n are some fixed functions in the Schwartz class such that $a_n(\sqrt{m}) = 1$ if $n = m$ and 0 if $n \neq m$ and similarly with $\hat{a}_n$. It is in the construction of this a_n that the modular forms are needed. Once we have the representation formula, the optimal properties of the magical function f of the Cohn-Elkies theorem essentially prescribe it. Different variants of this formula were obtained to address the universally optimality in dimension 8 and 24.

In this area, the more tantalizing problem that remains open is to prove that the hexagonal lattice in dimension two, that certainly is optimal for the spherical packing, is also universally optimal for many potentials. We still lack the corresponding Fourier interpolation formula and magical function f for this case. I am sure that Viazovska and her coauthors are working hard on this direction. ▷ Editorial

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The work of Duminil-Copin

by Guillem Perarnau (DMAT, **IMTech**)
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Last July the 2022 Fields Medals were announced at the International Congress (ICM) of the International Mathematical Union held online and in Helsinki, Finland. One of the four awardees is Hugo Duminil-Copin. The research of Duminil-Copin focuses on the understanding of deep physical phenomenon through probability theory. Duminil-Copin was awarded the prize for *solving longstanding problems in the probabilistic theory of phase transitions in statistical physics, especially in dimensions three and four*. His most impressive results provide a mathematically rigorous framework for several physical phenomena that were already observed by physicists decades ago.

Without any hesitation, one of his main achievements is the proof that the phase transition of the 3D Ising model is continuous at criticality [1], a joint work with Aizenmann and Sidoravicius in 2015. The *3D Ising model* is a mathematical model of magnetism on $\mathbb{Z}^3$ that is parameterized by a parameter $\beta > 0$ that can be understood as the inverse of the temperature. Magnetization can be measured as the long range correlation in the model. At high temperatures there is no magnetization and the typical configuration is random-like, while at low temperatures the model exhibits strong correlations and the typical configuration is highly structured. The existence of a critical temperature where magnetization emerges was already well-established in the literature, although the exact value is still unknown. Continuity at this value was proven for dimension two by Yang in 1952 [8], and for any dimension larger or equal than 4 by Aizenmann and Fernández in 1986 [2]. However, the 3D case remained elusive due to the lack of physical understanding. The contribution of Duminil-Copin and his coauthors was to show that, indeed, magnetization is continuous as the temperature goes through this critical temperature.

Another important contribution of Duminil-Copin has been the study of percolation models. Given the infinite planar grid $\mathbb{Z}^2$ and $p \in [0, 1]$, for each edge (and independently) flip a biased coin with probability p of turning heads and retain the edge if and only if the outcome is a head. This is known as *Bernoulli percolation* and can be easily analyzed due to its nice independence properties. A natural question is whether the percolated grid has an infinite connected component. Interestingly, there exists a 0-1 law for this event: depending on the value of p, almost surely, either there is one infinite component (and only one), or there is none. As the event of having an infinite component is monotonically increasing in p, one defines the *critical probability* p_c as the infimum p for which there is one such component almost surely. In 1980 Kesten [7] proved that $p_c = 1/2$. The problem is thus well-understood for Bernoulli percolation. In the 1970s, Fortuin and Kestelyn introduced a more complex percolation model known as the *random cluster model*: here the edges are not independently retained, but, a given configuration of edges appears with a probability depending (in an exponential way) on the number of edges and connected components it spans. The dependence on the number of edges is parameterized by $p \in [0,1]$ (as in the Bernoulli case) and the dependence on the number of components is controlled by a new parameter $q > 0$ (recovering the Bernoulli case when $q = 1$). Nonetheless, given any $q > 0$, the existence of an infinite component also follows a 0-1 law, is monotone (on p) and has a critical probability $p_c(q)$. In 2012, Beffara and Duminil-Copin [3] determined that $p_c(q) = \sqrt{q}/(1+\sqrt{q})$, also known as the *self-dual point* of $\mathbb{Z}^2$. Obtaining the value of the self-dual point is an easy computation which is based on the Euler characteristic formula.

Finally, we talk about one of Duminil-Copin's earliest contribution, obtained during his PhD, and that I particularly admire. Given a regular lattice it is straightforward to compute the number of walks of length $n \in \mathbb{N}$. A more interesting problem is to consider walks that do not self intersect. Namely, a *self avoiding walk* (SAW) of length n is a walk that starts at the origin, it has n steps, and it does not occupy any position more than once. SAWs have many applications, for instance they model the topological behavior of proteins or the boundary of random objects. While it is easy to see that there are exponentially many, computing the exact number (or even an asymptotic approximation) of SAWs of a given length in any lattice is a very hard problem. Duminil-Copin, together with his PhD supervisor Stanislav Smirnov (also a fields medalist), studied the problem in the hexagonal lattice $\mathbb{H}$, also known as the honeycomb lattice. Let h_n be the number of SAWs of length n in $\mathbb{H}$ and define the *connectivity constant* of the model as $c_{\mathbb{H}} = \lim_{n\to\infty}(h_n)^{1/n}$ (which exists as h_n is sub-multiplicative), that is the basis of the exponential growth of the sequence. They showed that $c_{\mathbb{H}} = \sqrt{2+\sqrt{2}} = 2\cos(\pi/8)$ [4]. Up to this date, this is essentially the only lattice for which the connectivity constant is known.

For further information about these and other research achievements of Duminil-Copin, we refer the interested reader to the recent surveys of Martin Hairer [6] and Geoffrey Grimmett [5] about his work. ▷ Editorial

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Combinatorics and Hodge Theory, after June Huh

by Anna de Mier (DMAT, **IMTech**) and
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"He Dropped Out to Become a Poet. Now He's Won a Fields Medal" was Jordana Cepelewicz' headline of her article in the Quantamagazine on July 5th, 2022. In her brilliant style, she emphasizes "his ability to wander through mathematical landscapes and find just the right objects —objects that he then uses to get the *seemingly disparate fields of geometry and combinatorics to talk to each other in new and exciting ways*. Starting in graduate school, *he has solved several major problems in combinatorics, forging a circuitous route by way of other branches of math* to get to the heart of each proof. Every time, finding that path is akin to a 'little miracle,' Huh said."

He was awarded the Fields Medal, the first in the field of combinatorics, "for bringing the ideas of Hodge theory to combinatorics, the proof of the Dowling–Wilson conjecture for geometric lattices, the proof of the Heron-Rota-Welsh conjecture for matroids, the development of the theory of Lorentzian polynomials, and the proof of the strong Mason conjecture."

In this note we try to substantiate these assessments by looking more closely into the mathematics through Huh's Fields lecture [1] at the ICM and the paper he published in the ICM Proceedings [2]. We also take into account Gil Kalai's ICM laudatio [3]. The rich historical developments that culminated in the breakthroughs of Huh and his collaborators are well documented in the works cited below, all easily accessible, and will not be our concern on this occasion. Note: a number within double square brackets refers to the reference in [2] labeled with that number, and they are linked to a pdf version whenever it is freely available.

In the first section we state the main combinatorial results, in the second we summarize the theory of Lorentzian polynomials, the third is devoted to Hodge theory, and in the last we look into examples of how this machinery works for solving conjectures in combinatorics that have been hitherto unreachable by other means. As we will see, the Hodge theory section is connected to the work of a number of Fields medalists: Michael Atiyah [[7]], Pierre Deligne [[10]], Alexander Grothendieck [[36]], Jean-Pierre Serre [[67]] and Shing-Tung Yau [[74]].

Combinatorial results. (1) Given a graph $G = (V, E)$, and a positive integer q, a *proper coloring* of G with q colors is a map $c : V \to [q]$ such that $c(a) \neq c(b)$ when $ab \in E$. The number of proper colorings of G with q colors turns out to be a polynomial in q (the *chromatic polynomial* of G) of the form

$$\chi_G(q) = a_n q^n - a_{n-1}q^{n-1} + \cdots + (-1)^{n-1}a_1 q,$$

where $n = |V|$ and $a_j \geqslant 0$ for $j = 1, \ldots, n$. The Read-Hoggar conjecture (1968, 1974) says that $a_1, \ldots, a_n$ is *log-concave*, which means that $a_j^2 \geqslant a_{j-1}a_{j+1}$ for $j = 2, \ldots, n-1$. This conjecture was proved by Huh in 2009 in his PhD research. The log-concavity implies that the sequence of coefficients is *unimodal* (this was Read's conjecture), which means that there is an index j such that $a_1 \leqslant \cdots \leqslant a_j \geqslant a_{j+1} \geqslant \cdots \geqslant a_n$.

(2) A *matroid* is a pair $M = (E, \mathcal{I})$, where E is a finite set and $\mathcal{I}$ is a family of subsets of E (called *independent sets*) that satisfy: (**i0**) the empty subset is independent; (**i1**) any subset of an independent set is independent; and (**i2**) if X, X' are independent and $|X| > |X'|$, then there exists $x \in X - X'$ such that $X' \cup \{x\}$ is independent. Thus a matroid is an abstraction of the notion of linearly independent sets of a finite set of vectors in a K-vector space (such matroids are said to be *representable* over the field K). It is also important to note

that a graph with edge-set E gives rise to a matroid by declaring a subset of edges independent if it contains no cycles. For a matroid $M = (E, \mathcal{I})$, the *rank* $r(X)$ of a subset X of E is defined by $r(X) = \max\{|X'| : X' \subseteq X, X' \in \mathcal{I}\}$. The *characteristic polynomial* of M, $\chi_M(q)$, is defined as $\chi_M(q) = \sum_{X \subseteq E}(-1)^{|X|}q^{r(E)-r(X)} = \sum_{j=0}^{d}(-1)^j a_j q^{d-j}$, where $d = r(E)$ and a_j is the number of subsets of E of rank j (the characteristic polynomial generalizes the notion of chromatic polynomial). The Heron-Rota-Welsh conjecture asserts that $a_0, a_1, \ldots, a_d$ is log-concave and it was proved in [[1]].

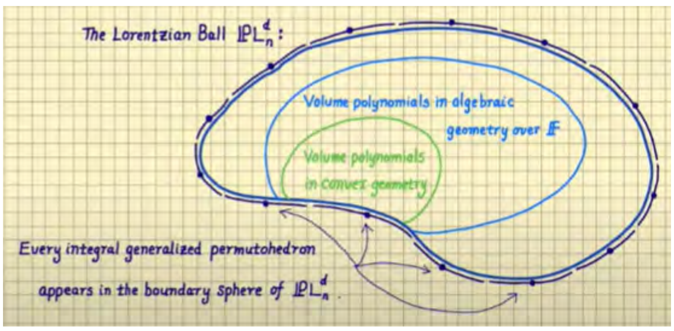
(3) Let $\mathcal{L}$ be a finite lattice, $r : \mathcal{L} \to \mathbb{N}$ its *rank* function, $\mathcal{L}^k = \{x \in \mathcal{L} : r(x) = k\}$, and $d = \dim(\mathcal{L})$ (the rank of its maximum element). We say that $\mathcal{L}$ is *geometric* if it is generated by $\mathcal{L}^1$ (the *atoms* of $\mathcal{L}$) and r satisfies the *submodular* property, namely $r(x) + r(x') \geqslant r(x \vee x') + r(x \wedge x')$ for all $x, x' \in \mathcal{L}$. The Dowling-Wilson *top-heavy* conjecture (1974) asserts that

$$|\mathcal{L}^k| \leqslant |\mathcal{L}^{d-k}| \text{ for all } k \leqslant d/2. \qquad (*)$$

Actually the conjecture was phrased for the lattice $\mathcal{L}(M)$ of flats of a matroid $M = (E, \mathcal{I})$ (a *flat* is a subset of E that is maximal for its rank) and it was proved in [[41]] (see also [[12]] for further enhancements). But this is not a more general statement than Eq. $(*)$, as the class of geometric lattices agrees with the class of lattices of flats of matroids.

(4) Let $i_k = i_k(M)$ be the number of independent sets of cardinal k in a finite matroid $M = (E, \mathcal{I})$. *Mason's ultra-strong conjecture* says that $i_k^2 \geqslant (1 + \frac{1}{k})(1 + \frac{1}{n-k})i_{k-1}i_{k+1}$, $n = |E|$. This conjecture was proved in [[17]]. As explained in the footnote 2 of [2], it was independently proved in the series [[2]], [[3]], [[4]].

Lorentzian polynomials. Let H_n^d be the space of real polynomials of degree d in n variables. The set of *Lorentzian polynomials* L_n^d is defined as follows. The elements of L_n^2 are specified by two conditions: (a_2) their coefficients are nonnegative, and (b_2) their signature (as quadratic forms) has at most one positive sign. For higher degrees d the set L_n^d is defined recursively by the following conditions: (a_d) $\partial_j f \in L_n^{d-1}$ for all $j \in [n]$, and (b_d) the set of (exponents of) monomials of f is the set of lattice points of an *integral generalized permutohedron* (that is, a polytope whose edges' directions have the form $e_j - e_k$, with $e_1, \ldots, e_n$ the standard basis of $\mathbf{R}^n$; for a reference on these objects, see [4]). One of the crucial results in [[17]] is that L_n^d is the closure of $\mathring{L}_n^d$, a set defined by the conditions: $(\mathring{a}_2)$ their coefficients are positive real numbers, $(\mathring{b}_2)$ their signature has exactly one positive sign, and $(\mathring{a}_d)$ $\partial_j f \in \mathring{L}_n^{d-1}$ for all $j \in [n]$.



Detail of slide number 13 of Huh's lecture at the ICM-22. Note the statement on the boundary sphere.

Theorem 2.28 of the same paper proves that the compact set $\mathbb{P}L_n^d \subset \mathbb{P}H_n^d$ is contractible, with contractible interior $\mathbb{P}\mathring{L}_n^d$, and conjectured that it is homeomorphic to a closed Euclidean ball (proved by Brändén [[16]]).

Hodge theory. The acclaimed proof techniques follow a pattern that was originated by Grothendieck in [[36]], where he raised two *conjectures on algebraic cycles* "in an attempt at understanding the conjectures of Weil on the ζ-functions of algebraic varieties". The first conjecture is analogous to "Lefschetz's structure theorem on the cohomology of a smooth projective variety over the complex field", and the second is a positivity statement "generalizing Weil's positivity theorem for abelian varieties; it is formally analogous to the famous Hodge inequalities, and is in fact a consequence of these in characteristic zero". Curiously enough, the final touch on the Weil conjectures was given by Deligne without relying on those conjectures, which to this day remain unproven, while we marvel at the scheme's fertility in domains that at a first glance may seem quite far apart and unrelated (but note that Weil's conjectures have a discrete character).

As presented by Huh, the scheme has three ingredients and three postulates (dubbed the *Kähler package* by Huh, for Kähler "first emphasized the importance of the respective objects in topology and geometry"). ***Ingredients***: (1) A graded real vector space $A = \bigoplus_{j=0}^{d} A^j$; (2) A convex cone K of graded linear maps $L : A^\star \to A^{\star+1}$ of grade 1; and (3) A symmetric bilinear pairing $P : A^\star \times A^{d-\star} \to \mathbf{R}$. ***Postulates***: For any $j \leqslant d/2$ and $L \in K$: (*Poincaré Duality*) $P : A^j \to (A^{d-j})^*$ is an isomorphism; (*Hard Lefschetz Propery*) $L^{d-2j} : A^j \to A^{d-j}$ is an isomorphism; (*Hodge-Riemann Relations*) The pairing $A^j \times A^j \to \mathbf{R}$, $(x, y) \mapsto (-1)^j P(x, L^{d-2j}y)$, is positive definite on the kernel of L^{d-2j+1} (the *primitive part* of A^j, to borrow a term from Lefschetz theory).

In the examples known so far, $A = A(X)$ depends on the objects X of some species. In the case of smooth projective varieties X, $A(X)$ is a cohomology ring and the pattern agrees essentially with Grothendieck's standard conjectures, which, as said, remain conjectures. The scheme works when X is a convex polytope and $A(X)$ its combinatorial cohomology [[45]], when X is an element of a Coxeter group and $A(X)$ its Soergel bimodule [[26]], and when X is a matroid, in which case $A(X)$ can be its Chow ring [[1]], its conormal Chow ring [[6]], or its intersection cohomology [[12]].

Fruits in the matroid field. The general strategy was summarized in slide number 14 of [1], while pointing out [[40]] and [[27]] for examples and conjectures for various X: (1) Given X, extract interesting multivariate generating functions from it; (2) Do we see any generalized permutohedra? (3) Do we see any Lorentzian polynomials? (4) Can we guess $A(X)$, $K(X)$, $P(X)$?

As in [1], let us end by describing how the Dowling–Wilson conjecture was solved. Given a geometric lattice $\mathcal{L}$ of rank d, consider the set J of its *bases*, that is, subsets of size d of $E = \mathcal{L}^1$ (the set of atoms) whose join has rank d. Then J is the set of lattice points of an integral generalized permutohedron, and hence the basis generating function $g = \sum_{\alpha \in J} w^\alpha$ is a Lorentzian polynomial. For illustrations, see the figure at the end.

Now define $\mathbf{H}(\mathcal{L}) = \{f : \mathcal{L} \to \mathbb{Q}\} = \bigoplus_{F \in \mathcal{L}} \mathbb{Q}\delta_F$ and make it a graded $\mathbb{Q}$-algebra (the *Möbius algebra* of $\mathcal{L}$) with the multiplication determined by

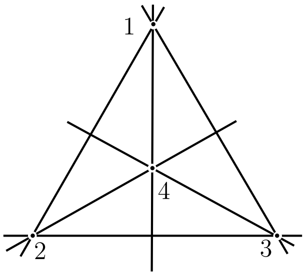
$$\delta_F \cdot \delta_{F'} = \begin{cases} \delta_{F \vee F'} & \text{if } r(F \vee F') = r(F) + r(F') \\ 0 & \text{otherwise.} \end{cases}$$

The *basis generating function* of $\mathcal{L}$ is $\frac{1}{d!}(\sum_{j \in E} w_j \delta_j)^d$. This suggests taking $A(\mathcal{L}) = \mathbf{H}(\mathcal{L})$; $K(\mathcal{L})$, the set of multiplications by positive linear combinations of the δ_j; and $P(\mathcal{L})$, multiplication in $\mathbf{H}(\mathcal{L})$ composed with $\mathbf{H}^d(\mathcal{L}) \simeq \mathbb{Q}$. But $\mathbf{H}(\mathcal{L})$

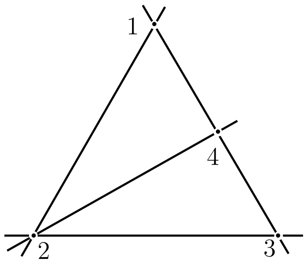
already fails to satisfy Poincaré duality, for $\dim \mathbf{H}^j(\mathcal{L}) = |\mathcal{L}^j|$ and in general $|\mathcal{L}^j| \neq |\mathcal{L}^{d-j}|$.

As shown in [[12]], the rescue from this failure came from the *intersection cohomology* of $\mathcal{L}$, $\mathbf{IH}(\mathcal{L})$, which is an indecomposable graded $\mathbf{H}(\mathcal{L})$-module endowed with a map $P : \mathbf{IH}(\mathcal{L}) \to \mathbf{IH}(\mathcal{L})^*[-d]$ that satisfies the following properties for every $j \leqslant d/2$ and every $L \in K(\mathcal{L})$: (*Poincaré duality*) $P : \mathbf{IH}^j(\mathcal{L}) \to \mathbf{IH}^{d-j}(\mathcal{L})^*$ is an isomorphism; (*Hard Lefschetz*) $L^{d-2j} : \mathbf{IH}^j(\mathcal{L}) \to \mathbf{IH}^{d-j}(\mathcal{L})$ is an isomorphism; and (*Hodge-Riemann relations*) The pairing $\mathbf{IH}^j(\mathcal{L}) \times \mathbf{IH}^j(\mathcal{L}) \to \mathbb{Q}$, $(x, y) \mapsto (-1)^j P(x, L^{d-2j}y)$ is positive definite on the kernel of L^{d-2j+1}. In addition, $\mathbf{IH}^0(\mathcal{L})$ generates a submodule isomorphic to $\mathbf{H}(\mathcal{L})$. The construction relies on the resolution of singularities of algebraic varieties.

Since the composition of $\mathbf{H}^j(\mathcal{L}) \hookrightarrow \mathbf{IH}^j(\mathcal{L})$ with the Hard-Lefschetz isomorphism $\mathbf{IH}^j(\mathcal{L}) \simeq \mathbf{IH}^{d-j}(\mathcal{L})$ is injective, it follows that $L^{d-2j} : \mathbf{H}^j(\mathcal{L}) \to \mathbf{H}^{n-j}(\mathcal{L})$ composed with $\mathbf{H}^{d-j} \hookrightarrow \mathbf{IH}^{d-j}(\mathcal{L})$ is injective and consequently $L^{d-2j} : \mathbf{H}^j(\mathcal{L}) \to \mathbf{H}^{d-j}(\mathcal{L})$ is injective, which proves that $|\mathcal{L}^j| \leqslant |\mathcal{L}^{d-j}|$.



$g = w_1w_2w_3 + w_1w_2w_4 + w_1w_3w_4 + w_2w_3w_4$



$g = w_1w_2w_3 + w_1w_2w_4 + w_2w_3w_4$

The proper non-trivial flats are the points and the lines. Bases are minimal sets of points that affinely span the plane. ▷ Editorial

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Barry Mazur Awarded 2022 Chern Medal

by Joan Carles Lario (DMAT)

Received on November 27th, 2022

This year the Chern Medal has been awarded to Professor Barry Mazur of Harvard University. This medal is awarded every 4 years during the celebration of the International Congress of Mathematics "to an individual whose accomplishments warrant the highest level of recognition for outstanding achievements in the field of mathematics".

Barry Mazur has received many awards, distinctions, and medals throughout his life. However, he prefers to solve specific problems rather than collecting honors, being recognized, or pursuing media attention. This time he should feel more comfortable since 50% of the cash prize consists of distributing it to institutions that promote mathematics. Indeed, this is the style and personality of Barry Mazur, always avoiding being in the spotlight. He prefers to share his mathematical ideas generously that constantly germinate from his endless creativity.

As a teenager Barry Mazur was attracted by the magic of the airwaves and became a radio amateur. Considering himself as a "philosopher of electronics", he was eager to learn what was behind these remote actions and eventually stumbled upon mathematics. At the end of his studies at MIT, he did a thesis where he solved the topological Schoenflies problem, which generalizes the Jordan curve theorem for higher dimensions.

Because of Barry Mazur's contributions to Knot Theory, he established a conversation with his department colleague David Munford at the Science Center. This conversation led him to set an analogy between prime numbers and knots. In this way, Arithmetic Topology was born.

Barry Mazur had already begun to explore Arithmetic Topology, but thanks to a "homework" assignment by Alexandre Grothendieck, he began to set important roots in Arithmetic Geometry. The article *Modular Curves and The Eisenstein Ideal* was a first result in this field. A 70-year-old problem on the possible structures of the torsion groups of elliptic curves defined over the field of rational numbers was finally solved. For this article published in 1977, Barry Mazur received the Steele Prize, but —more importantly— this work contains the foundations that gave rise to many other results. For example, the proof of the Iwasawa conjecture (together with Andrew Wiles in the mid-1980s); the launching of Galois deformation theory (which ultimately allowed the proof of the famous Fermat Last Theorem by Andrew Wiles et al. in the mid-1990s), among others. Today, Barry Mazur continues to explore new uncharted arithmetic territories and develop new research avenues, such as Diophantine Stability.

Barry Mazur is much more than a mathematician. He is an artist who uses mathematics as a creative tool, and in the same way he relates to literature, music, physics, philosophy, law, painting, etc. His multi-disciplinary courses at Harvard are famous. One might also say that Barry Mazur is a creator who has always dedicated himself to ignite the creativity of new generations. He always relates to his students (more than 60 supervised doctoral theses) and colleagues in a simple and affable way, with an optimism and generosity beyond the usual standards.

Barry Mazur's relationship with the Barcelona Number Theory (UB-UAB-UPC) research group has been very productive and gratifying for the last 30 years. We were able to enjoy his company and mastery during his two visits to Catalonia and reciprocally he has always welcomed us in his home. On occasion of his 60th birthday, during the congress organized in Boston, we gave him a "porró" ("pooh'Ro", a Catalan topological bottle), a bottle of wine, and a card containing a congratulatory phrase that we think is very appropriate to repeat now: "Barry, thanks for being such a human being". ▷ Editorial

IMTech Colloquium 9/11/2022

by Gemma Huguet (DMAT, **IMTech**)

Received on 30 November, 2022.

On November 9, 2022, Professor Sebastià Xambó delivered the **IMTech** Colloquium Lecture at the Faculty of Mathematics and Statistics (FME). He is Emeritus Full Professor of Information Theory and Coding at the UPC Mathematics Department (DMAT) and is a member of **IMTech**. He has been president of the Catalan Mathematical Society (1995-2002) and dean of the FME (2003-2007). In 2019 he was awarded the Royal Spanish Mathematical Society Medal (RSME) and in 2020 the Catalan

Government distinguished him with a Narcís Monturiol Medal.

In the last few years, he has been interested in algorithmic learning, its mathematical foundations, and its application to mathematical research. In that regard, he organized a mini symposium at ICIAM-2019 (Valencia) on *Systems, Patterns and Data Engineering with Geometric Calculus* and edited a volume with the same title published in 2021 in the Springer ICIAM 2019 Series. He has been visiting the BSC/CNS since 2019, within the group of High Performance Artificial Intelligence, and is co-organizing a special session at ICCA 2023 on *Data science in mathematics, physics and engineering*.

The title of his lecture was *Artificial intelligences and mathematics*. In it he overviewed various forms of algorithmic learning, with emphasis on those based on artificial neural networks, their current achievements, their mathematical underpinnings, and their likely trends in the next few years. He also addressed the deep tensions inherent to polarities such as brain-mind, biological-artificial, scientific-industrial, academic-corporative, and their inevitable impact on the understanding of human nature and society.

In the last part of the talk he presented some of the ideas that seem more promising to move significantly closer to effective models of intelligence, particularly with the approaches of Karl Friston and his school (Bayesian methods based on the principle of minimum expected free energy) and of Stephen Grossberg and collaborators (after his recent treatise *Conscious Mind, Resonant Brain: How each brain makes a mind*) and the roles that mathematics may play in this evolution. Along the talk he reviewed and presented an extensive bibliography, for those interested in expanding the ideas discussed.

The slides of the talk are available at AIs&Math and the video recording of the talk is available at Zona video.
▷Editorial

IMTech INIREC scholarships

by Gemma Huguet (DMAT, **IMTech**)
Received on 30 November, 2022.

In January 2022, **IMTech** awarded 4 scholarships INIREC (Iniciació a la Recerca/Initiation to Research) to UPC master students that developed a research project within the research groups of the **IMTech**. The selection committee was formed by professors Marc Noy (DMAT, **IMTech** director), José Muñoz (DMAT, **IMTech** secretary), Marta Casanellas (DMAT, **IMTech**), Pedro Delicado (EIO, **IMTech**) and Pau Martín (DMAT, **IMTech**).

The awarded candidates were Beatriz Barbero Lucas, Javier Guillán Rial and Andrea Suárez Segarra, who were students of the UPC Master in Advanced Mathematics and Mathematical Engineering (MAMME) and Ali Kalaban Ammar, who is a student of the Master's degree in Numerical Methods in Engineering.

Beatriz Barbero Lucas holds a Bachelor's degree in Mathematics from the University of Salamanca and she was enrolled in the MAMME. Currently she is working for a PhD at the University College Dublin in the field of Algebraic Geometry and Cryptography.

During her INIREC scholarship, she worked on her master thesis *Rings of differential operators on singular varieties* under the supervision of Professor Josep Álvarez (DMAT, **IMTech**), which was defended in July 1st, 2022.

Summary: The project is centered in the study of rings of differential operators on singular varieties and the theory of D-modules. A D-module is defined as a module over the ring of differential operators on a k-algebra. We start with the case of Weyl algebras which coincide with the ring of k-linear differential operators on a polynomial ring $k[x_1, \dots, x_n]$. We prove that this ring of differential operators is a simple Noetherian domain whose dimension is two times the dimension of the polynomial ring. Also, considering modules over this ring we show Bernstein's inequality that gives us a lower bound to the dimension of the module and, using this result, we define holonomic D-modules. Our goal is to study what good structural properties of the regular case can be extended in the singular one. We use the results proved in the case of Weyl algebras to give a description of the ring of k-linear differential operators on a finitely generated k-algebra that can always be presented as a quotient $S = R/I$ of a polynomial ring $R = k[x_1, \dots, x_n]$ by an ideal I. We prove that those differential operators can be obtained in terms of the differential operators that preserve the ideal and differential operators on the Weyl algebra. And although we show that the ring of differential operators of a finitely generated k-algebra does not have properties as good as Weyl algebras, however we obtain some of them under several conditions. We study modules over this kind of rings of differential operators and their dimension, proving a generalized Bernstein's inequality under some conditions. Finally, we apply these results to study the case of the ring of differential operators on a hyperplane arrangement and explain several methods to obtain a system of generators of this ring.

Javier Guillán Rial completed his undergraduate studies in Mathematics and Physics in the University of Santiago de Compostela and his master's degree within the MAMME. He is mostly interested in Number Theory and Commutative Algebra, and he is currently doing a PhD in the University of Barcelona in the field of Galois representations.

During his INIREC scholarship, he worked on his master thesis *Hecke Algebras, Deformation Rings and Singularities* under the supervision of Professor Víctor Rotger (DMAT, **IMTech**, CRM), which was defended in July 7th, 2022.

Summary: The aim of this work is to cover the main background of Hecke algebras and deformations of Galois Representations, as well as the main results on complete intersection rings used in the most important results on Number Theory (for example, Fermat's Last Theorem). This thesis is thought to introduce comprehensively (assuming certain knowledge on commutative algebra and modular forms) the basic results on the structure of Hecke algebras and the explicit construction of the Universal Deformation Rings associated to certain Galois representations.

Andrea Suárez Segarra completed the Bachelor of Mathematics at the École Polytechnique Fédérale de Lausanne, in Switzerland. She continued then her studies at the UPC in Barcelona, where she recently finished the MAMME. Currenlty, she is looking for projects where her mathematical knowledge can be used for the benefit of environmental and social issues.

During her INIREC scholarship, she worked on her master thesis *Oscillatory dynamics with applications to cognitive tasks* under the supervison of Professor Gemma Huguet (DMAT, **IMTech**, CRM), which was defended in October 18th, 2022.

Summary: Oscillations are ubiquitous in the brain and robustly correlate with distinct cognitive tasks. A specific type of oscillatory signals allows robust switching between states in networks involved in memorizing tasks. In particular, slow oscillations lead to an activation of the neuronal populations whereas oscillations in the beta range are effective in clearing the memory states. In this master thesis, previous works are revisited in order to provide a detailed analysis of the mechanisms underlying the states' switching and their dependence on the network parameters. The model studied is a macroscopic description of the network recently derived due to mean-field theory advances. The role of spiking synchrony in the "switching off" of the active states is identified by means of bifurcation analysis and the study of the fixed points under the stroboscopic map. Finally, we propose an application of the effect of oscillations in a context of working memory.

Ali Kalaban Ammar studied a bachelor's degree in mechanical engineering at the Lebanese American University (Lebanon), and he is currently a second year student in the Master's degree in Numerical Methods in Engineering at UPC.

During his INIREC scholarship, he worked on a project entitled *Computational modeling of the electromechanical behavior of epithelia* under the supervision of Professor Pablo Sáez (DMAT, **IMTech**).

Summary: Epithelia are tissues that cover most internal and external compartments of the body. Epithelial cells are polarized, connected to each other with cell junctions and supported by an extracellular matrix layer. The electrochemistry of the cell is a main decider of the function of the cell and its volume, electrochemistry is managed by ion channels, chemical and electrical balance. In this paper we model the electrochemistry of an airway epithelial cell which has periciliary fluid (PCL) at the apical membrane that must be maintained at a certain depth, if not mucociliary transport would cease.
▷ Editorial

First meeting of the RandNET project

by Marc Noy (DMAT, **IMTech**) and
Lluís Vena (DMAT, **IMTech**)
Received on December 21, 2022.

Randomness and Learning in Networks (RandNET) is a EU project funded by the Marie Skłodowska-Curie Research and Innovation Staff Exchange Programme (RISE) from 2021 to 2025. It brings together researchers from the areas of combinatorics, probability theory, computer science and statistics with the aim of blending approaches from these areas in the rigorous mathematical foundations for analysing random networks. A second goal of the project is to establish a wide platform of knowledge dissemination on the topic of randomness and learning in networks for use of specialists from all scientific disciplines. There are 14 participanting institutions in RandNET: UPC (coordinating node), U. Oxford, É. Polytechnique, T.U. Vienna, T.U. Eindhoven, U. Pompeu Fabra, Charles U. Prague, U. Paris Cité, NOKIA/Bell Labs France, Georgia Tech, U. Chile, IMPA Rio de Janeiro, McGill U., and UC San Diego.

Besides the exchange of researchers for advacing in the goals of the project yearly RandNET meetings are planned. During August 22-30, 2022 took place the first meeting at T.U. Eindhoven, the RandNET Summer School and Workshop on Random Graphs, funded by the RandNET grant and the Eurandom workshop centre in Eindhoven. It was organized by Serte Donderwinkel and Christina Goldschmidt (Oxford), and Remco van der Hofstad, and Joost Jorritsma (Eindhoven). There were about 90 participants, and about 60 were either PhD students or postodoctoral researchers, most of them receiving financial support. The program was organized as follows.

1. Four courses on Random Graphs, each consisting of a total of four sessions of 75 minutes: *Phase transitions in random graphs*, by Mihyung Kang; *Percolation on finite transitive graphs*, by Tom Hutchcroft; *A quick tour of random maps*, by Grégory Miermont; and *Scaling limits of random graphs*, by Minmin Wang.
2. Three one hour invited talks. *Synchronizing Words in Random Automata*, by Guillaume Chapuy; *Split trees: A unifying model for many important random trees of logarithmic height*, by Cecilia Holmgren; *Correlated stochastic block models: graph matching and community recovery*, by Miklos Racz. And five one-hour-sessions of Contributed Talks from the participants, each with four 15-minutes presentations.
3. Workshop on Open Problems. The afternoons were devoted to work on some of the 35 open problems proposed by the participants. In the first afternoon session each problem was given a 5 minutes slot so that the proposer could pitch it for the rest of the participants. After the participants sent their preferences they were split into 15 groups of between 3 to 6 persons, and the organizers assigned a problem to each of those groups. The problems were selected trying to preserve the preferences while maintaining a minimal quorum on the number of persons working on each problem. The groups combined participants in different stages of their careers: each group had some senior participant and/or the participant proposing the problem, and a mixture of postdoctoral researchers and Ph.D. students at different points of their studies. Each of the other seven afternoon sessions started with two parallel one-hour-long sessions where each group reported on their progress, or the lack thereoff, from the previous day; exposed some changes on the problem, according to the difficulties or new findings, either in the literature or due to the work that they were developing; observed what the other groups were working on; and gave feedback and comments on the work of other groups. The parallel sessions were changing the group composition, so that everyone could follow the progress from the rest of the participants and give feedback or new ideas. The afternoon sessions were completed with 3 to 4 hours working sessions on the open problems.

After the workshop the groups have continued the research projects and we expect that several publications will come out in the near future. The format of the meeting showed to be very effective for the training of young researchers and for the interaction among the various research groups associated to RandNET. The next meetings will take place in Prague (2023), Rio de Janeiro (2024) and Vienna (2025). ▷ Editorial

Jaume Peraire, H. N. Slater Professor of Aeronautics and Astronautics at the Massachusetts Institute of Technology (MIT), Doctor *Honoris Causa* at UPC. by Antonio Huerta (LaCàN, IMTech)

Received on 13 November, 2022.

On June 3rd 2022 the Universitat Politècnica de Catalunya (UPC) conferred to Jaume Peraire an honorary doctorate, the highest recognition in life to those who have stood out in their career for their work, virtue, merits and actions, and who are not doctors by the university that bestows the degree (booklet of the ceremony). He is a former UPC student who has built up an exceptional academic career and, in spite of this, has never forgotten his alma mater, supporting and helping unconditionally our academic community whenever requested.

Academia rests on three essential pillars: research, education and service. They are universal, all three are intertwined, all are utterly necessary, and none is more important than the others. Professor Peraire has excelled in those three dimensions and, to top it off, he has done it spilling tons of passion, curiosity and joy to undergraduate and graduate students, co-workers and colleagues. Also, key ingredients for success.

It is important to stress that the research area of Professor Peraire requires a non-trivial duality. On the one hand, it requires a thorough knowledge of (and many contributions in) mathematics in order to understand the essence of the methods and to propose novel advances. On the other hand, engineering applications are the "driving force" of this research and therefore it is crucial to keep in mind the applicability of the proposed strategies and methods in relevant engineering problems. This duality, mixing rigor and application, has driven his academic career. This interdisciplinarity is not an easy path but its contributions overcome relevant and pertinent societal challenges, extend the frontier of knowledge and allow applying these advances in educating the new generation in order to build a better society (i.e., a fairer society). Mathematics and Engineering are a winning combination!

Professor Peraire research, education and service has been devoted to Computational Science and Engineering. This is a relatively new and emerging discipline that is concerned with the development and application of computational models and simulations (mathematical models, numerical methods, high-performance algorithms, scientific machine learning, etc.). These strategies are designed to solve complex physics problems (in many occasions characterized by multiscale multiphysics dynamics) that appear in science and engineering analysis and design. The key objective is to empower decision-makers with solid, provable and certifiable (when possible) strategies to solve engineering problems. Thus, helping to reduce the uncertainties in arriving to “a good solution”, recall the citation: “engineering problems are under-defined, there are many solutions, good, bad and indifferent. The art is to arrive at a good solution. This is a creative activity, involving imagination, intuition, and deliberate choice.” Sir Ove Arup (1895-1988). Mathematics and mathematical modeling are the cure for those uncertainties. This is the demanding field where Professor Peraire stands out. ▷Editorial

Reviews

Papers

Interpolation of Brill-Noether curves, by Eric Larson and Isabel Vogt. arXiv (v1, 24 Jan 2022; v2, 5 May 2022). Reviewed by Sebastià Xambó.

This is a landmark work in algebraic geometry. A popular account, with touches on the authors and their human and academic tracks, was published in a Quanta article by Jordana Cepelewicz. In this note we will look a bit more closely on the mathematical achievements and their significance as a culmination of a long and rich chain of contributions by many authors.

Although it is 71 pages long, its exploit is described in its most brief abstract: *We determine the number of general points through which a Brill-Noether curve of fixed degree can be passed.* The points are supposed to lie in a projective space $\mathbf{P}^r$ ($r \geqslant 2$). Curves are subvarieties of dimension 1 of $\mathbf{P}^r$ and for any irreducible family of curves we may ask how many general points can a general curve of the family go through. A naive count is that for a curve in $\mathbf{P}^r$ to pass through a point imposes $r-1$ conditions and if the points are in general position, these conditions are expected to be independent, and hence this predicts $n = \lfloor D/(r-1) \rfloor$ points, where D is the dimension of the family. For example, for rational curves of degree d in the plane, we have $D = 3d-1$, which in this case agrees with n (5 points for conics, 8 points for rational cubics, and so on). In what follows, the integers d, g and r are assumed to satisfy $d \geqslant 1$, $g \geqslant 0$, and $r \geqslant 2$.

The definition of the Brill-Noether (BN) family of curves in $\mathbf{P}^r$ used by the authors is based on two facts (see the paper for references): (1) If C is a curve of genus g, then there exist non-degenerate maps $C \to \mathbf{P}^r$ if and only if $\rho(d,g,r) := (r+1)d - rg - r(r+1) \geqslant 0$; and (2) The universal space of such maps has a unique component dominating $\overline{\mathcal{M}}_g$ (the compactification of the moduli space $\mathcal{M}_g$ of curves of genus g). A BN curve is then defined as a stable map $f : C \to \mathbf{P}^r$ corresponding to a point in that component and for such curves the authors prove the following theorem (1.2 in the paper): *If $\rho(d,g,r) \geqslant 0$, then there is a BN curve of degree d and genus g through n general points in $\mathbf{P}^r$, if and only if* [∗] $(r-1)n \leqslant (r+1)d - (r-3)(g-1)$, *except for the cases* $(d,g,r) \in \{(5,2,3),(6,4,3),(7,2,5),(10,6,5)\}$. Note that the relation [∗] yields the expected value of n because the family of BN curves has dimension $D = (r+1)d - (r-3)(g-1)$.

The notion of passing through general points is a special case of being incident to linear varieties. To deal with this more general situation, the authors introduce a (cohomological) condition on the normal bundle N_C (that they call *interpolation property*) which implies that *there is a BN curve of degree d and* *genus g incident with general linear spaces L_j of dimension ℓ_j if and only if* [$**$] $\sum(r-1-\ell_j) \leqslant (r+1)d-(r-3)(g-1)$.

For BN curves C such that N_C has the interpolating property the authors prove the following theorem (1.4 in the paper): *If $\rho(d,g,r) \geqslant 0$, and C is a general BN curve in $\mathbf{P}^r$, then N_C satisfies the interpolation property if and only if neither of the following hold:* (1) $(d,g,r) \in \{(5,2,3),(6,4.3),(6,2,4),(7,2,5),(10,6,5)\}$, *and* (2) *the characteristic is* 2, $g=0$, *and* $d \not\equiv 1 \mod r-1$. Note that the triples (d,g,r) in (1) are those in the first theorem with the addition of $(6,2,4)$. The maximum possible n for these exceptional cases is 10, 12, 9, 10, 12, respectively, and the fact that they cannot be achieved is argued case by case with clever geometric reasoning (§2,1). In the case $(6,2,4)$ the curves behave as expected for points, as it is not an exception to the first theorem, but they do not for incidence to linear spaces, as [$**$] is satisfied for 9 points and one line but it turns out that such a curve does not exist. The argument to see this is similar for the cases $(5,2,3)$, $(6,2,4)$ and $(7,2,5)$, which have the form $(r+2,2,r)$, and is based on the fact that C is contained in a family of scrolls S of dimension $\delta=r^2+2r-6$. In the case $(6,2,4)$, for example, $\delta=18$, and a general S cannot go through 9 points in $\mathbf{P}^4$ (18 conditions on S) and meet one line (1 condition on S), and so neither can C.

The proof techniques use a method of degeneration to reducible curves $X \cup Y$, that includes reconstructions of $N_{X\cup Y}$ from N_X, N_Y and data at $X \cap Y$. The method (see [2] for an early advance) is used recursively, and the whole outcome is like a tapestry of great complexity with very skillful geometric arguments in each strand. Although it is not a computer based proof, the Python code in the Appendix A is a handy tool to manage lengthy computations. In fact, as a reviewer it has been helpful to write Python functions rho(d,g,r), to check the condition ρ, and BN(d,g,r), to deliver the maximum n satisfying [$*$].

Let me finish by pointing out the main historical inputs for this outstanding work (cf. [1]). The condition $\rho(d,g,r) \geqslant 0$ that first appears in the definition of the BN curves is the *Brill-Noether conjecture* and it was proved by Kleiman and Laksov [3], Griffiths and Harris [4], and Gieseker [5]. The degeneration techniques developed in the present paper complete earlier works, especially [6]. In [7] and [8], interpolation is solved for $r=3$ and $r=4$, respectively. These kind of methods were also applied by Erik Larson [9] to prove Severi's maximal rank conjecture. If the techniques and results of the Larson-Vogt paper fully answer an important question about BN curves, it is nevertheless clear that they also open new avenues for research. ▷ Editorial

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A Proof of the Kahn-Kalai Conjecture, by Jinyoung Park and Huy Tuan Pham, [4]. Reviewed by Jordi Castellví and Miquel Ortega.

A *random graph* on n vertices is a random sample from the edges of the complete graph K_n. In the commonly used Erdös-Rényi model, these are sampled independently and with uniform probability $p=p(n)$. Since its birth, one of the central topics in the study of random graphs has been the research of *threshold* phenomena. Perhaps surprisingly, for most interesting properties of a graph there exists a threshold probability $p^*=p^*(n)$ such that the property holds asymptotically almost surely when $p/p^* \to \infty$ and does not hold when $p/p^* \to 0$. For example, with probability tending to one, $G(n,p)$ contains a triangle as a subgraph for $p=\omega(1/n)$ and contains no triangle for $p=o(1/n)$.

Threshold probabilities have been extensively studied for many properties, such as the appearance of a Hamiltonian cycle or the existence of a perfect matching. For many examples of interest, a straightforward application of Markov's inequality will give a lower bound on the threshold probability depending on the expected value of the property. However, improvements on this lower bound or work on the upper bound have required until now a mostly case by case analysis.

Jeff Kahn and Gil Kalai [3] conjectured that the probability threshold for such examples is in fact close to the lower bound obtained by doing the simple expectation estimates. Precisely, they conjectured that the true probability threshold differs from the *expectation threshold* at most by a logarithmic term. In order to do so, in [3] they recast the problem in somewhat more general terms of increasing families of sets so as to rigorously state what the expectation threshold is. Note that this logarithmic term is necessary for the conjecture to be true, since there are examples, such as the appearance of a Hamiltonian cycle, where it is reached.

Until very recently, this conjecture seemed out of reach and it was not even clear whether one should try to prove it or look for a counterexample. In the reviewed article, Park and Pham give a beautiful and unexpected five-page-long proof of the conjecture. The authors were inspired by some important results that had appeared during the previous years, mainly the proof of the *fractional* version of the conjecture [2] and the breakthrough on the Sunflower Lemma [1], a seemingly unrelated topic on which [2] was also based. In order to learn from these new tools, the UPC GAPCOMB research group organized this fall a Reading Seminar dedicated to the study of Park and Pham's proof and of the results that built up to it.

Not only does their accomplishment vastly simplify the proofs of existing results, but it also provides a new and powerful technique which is likely to be useful for other problems.

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Events

FME: Graduation ceremony of the 2021-2022 promotions

On December 21, 2022, the UPC's Vertex auditorium hosted the graduation ceremony for the 2021-2022 degrees offered by the UPC's Faculty of Mathematics and Statistics (FME). At the end of each academic year, it is one of the most cherished celebrations organized by the Faculty in appreciation of their graduates and their families.

The graduates belonged to the undergraduate degrees in Mathematics and in Statistics (this is a joint UB-UPC degree); to the master's degrees in Advanced Mathematics and Mathematical Engineering (MAMME) and in Statistics and Operations Research (MESIO, joint UPC-UB); and to the doctorates in Applied Mathematics and in Statistics.

The ceremony began with the welcome words by the dean of the FME, Jaume Franch, who stressed that this event marked the end of the celebration of the faculty's 30 years, and was followed by the speeches of the godparents of the promotion: Professor Xavier Puig for the Statistics degrees and Professor Tere Martínez-Seara for the Mathematics degrees.

The awarding of the diplomas was one of the most special moments of the day, where the real protagonists of the event, the graduates, went on stage to collect their recognition and enshrine the moment with a photo. The speeches of two graduates, in representation of their peers, was another memorable special moment.

The theses defended during the academic year were cited (nine in Applied Mathematics and one in Statistics and Operations Research) and special prizes, sponsored by various companies and institutions, were awarded to the best academic records.

The closing was officiated by the vice-rector of Quality and Linguistic Policy of the UPC, Professor Imma Ribas and ended with a luxury interpretation of the *gaudeamus igitur* by the FME's student choir "Cor Ol·lari".

SCM: New Board of Directors

On November 30, 2022, the Council of the Catalan Mathematical Society (SCM) renewed its Board of Directors for a four year period. It is now constituted as follows: Montserrat Alsina, President (picture); Joan Porti and Josep Vives, Vice-presidents; Albert Granados, Treasurer; Margarida Mitjana, Secretary, and David Virgili and Clara Mateo, members.

We are grateful to this new board for their willingness to steer the SCM and thus serve the mathematical community, and wish them success in their commitment. We also thank the preceding board, chaired by Dolors Herbera, for the achievements during their mandate. ▷ Editorial

CFIS: Graduation ceremony 2022

The graduation ceremony of the fourteenth promotion of the UPC Higher Interdisciplinary Training Center (CFIS) was held in the UPC Vertex Auditorium on the 22nd of December, 2022. The authorities in the presidential table were Daniel Crespo (UPC rector), Miguel Ángel Barja (CFIS director), David Marín (president of the CFIS Board of Trustees), Francisco Javier Navallas (godfather of the promotion), and Teresa Nieves Chinchilla (Research Astrophysicist–Project Scientist for the Solar Orbiter Collaboration, NASA-Goddard Space Flight Center).

The first part of the ceremony began with the greeting and address by the CFIS director under the motto, after two academic years of Covid, *The return to normality*; it was followed by the speech of the president of the CFIS Board of Trustees, and ended with the magnificent lecture on *Exploring the Sun's atmosphere* by Teresa Nieves Chinchilla.

Then there was a musical interlude in which Marc Torrecillas, a CFIS student, offered an excellent interpretation of Chopin's *Raindrop prelude*.

In the second part the newly admitted students received their accreditation diplomas, the CFIS-2022 prize to the best academic record (sponsored by CFIS, Esperanto Technologies Europe and HP) was awarded, and the diplomas to students who have obtained their UPC Higher Interdisciplinary Engineering degree in the 2022 academic year were delivered, and representatives of the 2022 class adressed the audience by sharing their experiences through their studies. Finally, the godfather of the promotion pronounced his oration and the rector's speech closed the event.

From left to right: Francisco J. Navallas, Miguel Á. Barja, Daniel Crespo, David Marín, and Teresa Nieves Chinchilla.

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The *Seepferdchen* image in the cover, from Herwig Hauser's Gallery of Singular Algebraic Surfaces, is one of those included in the Imaginary exhibits.