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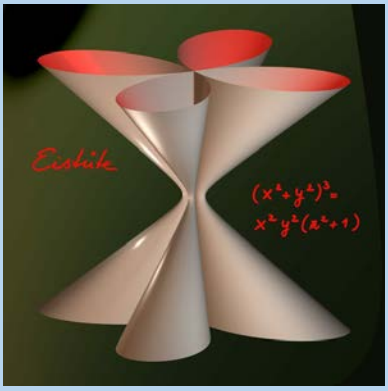

IMTECH Newsletter 7

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IMTech Newsletter

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Contents

Editorial

Starting 2024 this Newsletter is published yearly instead of biannually.

Professor Sebastià Xambó, who has served as Editor-in-chief of the IMTech Newsletter since the beginning in 2021, has now retired from UPC. Prof. Sebastià Xambó has been the driving force behind this project. We wish to express our warmest thanks for his superb and generous task during these years, and we wish him all the best in this new period. We are left with the challenge of trying to maintain the high standards he set for this publication.

IMTech members, Toni Guillamon, José J. Muñoz, and Víctor Mañosa have been recently appointed as Full Professors of the DMAT. On the occasion of this happy event, the NL held interviews with the new Professors: see ▷T. Guillamon, ▷J. Muñoz, and ▷V. Mañosa. They were formally appointed by the UPC Rector on 10th of June of 2024 (J. Muñoz) and 12th of November of 2024 (A. Guillamon and V. Mañosa).

Next we have two Research focus notes, three PhD highlights and three Outreach contributions. Simeon Ball presents *Quantum error-correcting codes and their geometries* [▷] and Marino Arroyo and Pablo Sáez present their work with co-authors on *The laminin–keratin link shields the nucleus from mechanical deformation and signalling* [▷]. Then follow the thesis summaries of Waleed Mirza [▷], on *A theoretical and computational study of the active self-organization of nematic patterns in thin cytoskeletal layers and their effect on curvature*; of Ashutosh Bijalwan [▷] on *Numerical optimisation of worm locomotion on frictional substrates*; and of Joaquim Brugués [▷], on *Floer homology for b-symplectic manifolds*.

The Outreach contributions are by Jezabel Curbelo [▷] on *Research that changed the dynamic of fluids turns 100*; Pablo Sáez [▷] with the title *How the transmission of forces between cellular networks explains the expression of transcriptional regulators* and Agustín Moreno [▷] on *The circular restricted three-body problem: a modern symplectic viewpoint*.

In the Chronicles section, we present a report of the Abel Prize 2024 awarded to Michel Talagrand, by professor Gábor Lugosi (▷). We also include reviews of the work of some of the ECM Prize winners awarded in the 9the European Congresss of Mathematics which took place in Sevilla 15-19 of July, 2024. The contributions about the ECM prizes are by Michelle Dolce and Xavier Fernández Real [▷], on Maria Colombo ; by Juanjo Rué [▷] on Tom Hutchcroft ; by Pablo Candela [▷] on Frederik Manners ; by Tássio Naia [▷] on Richard Montgomery and by Joaquim Ortega Cerdà [▷], on Canylo Radchenko .

In the Events section we report on the activities organized by IMTech during the year 2024. One was the IMTech Spring Colloquium on April 10, 2025. The invited speaker was professor Marcel Guàrdia (Universitat de Barcelona), who lectured on *Unstable motions in Celestial Mechanics*. As a novelty, this year we had the 1st IMTech Meeting, which took place on October 28th on the occasion of the hybrid meeting of the IMTech scientific Advisory Board. We had the presential visit of committe members Daniel Peralta (ICMAT) and Oleg Pikhurko (University of Warwick, Mathematics Institute and DIMAP). We include chronicles of these events, the Spring IMTech colloquium [▷] and the 1st IMTech meeting [] by Gemma Huguet. Finally we present a report by Marc Noy [▷] on the workshop Interplays between algebra, combinatorics and proof formalization.

Interviews

José J. Muñoz is Full Professor at the Department of Mathematics (**DMAT**), member of **IMTech**, and member of the research group Mathematical and Computational Modelling (LaCàN).

He received his degree in Mechanical and Civil Engineering in 1998 from UPC and École Centrale Paris. After two years working as structural engineer, he enrolled an completed his PhD at Imperial College London in 2004 under the supervision of Prof. Mike Crisfield and Gordan Jelenić. He joined UPC in 2006 with a Juan de la Cierva grant, and has been a lecturer (2008) and Associate Professor (*agregat*, 2011) also at UPC. He has been secretary of IMTech since its creation in 2020.

Dr. Muñoz has a background in non-linear dynamics, contact mechanics and materials stability. He currently develops computational models for cells, biological tissues and organisms, and in particular has applied the methods in embryogenesis, wound healing, tissue rheology and cell migration. He has supervised six PhD theses and has been PI of six projects.

NL. ***Congratulations on your recent appointment as Full Professor in Applied Mathematics! Could you share when your interest in mathematics first began, and how it developed throughout your undergraduate studies?***

During my engineering studies I was fascinated by the ability of models to simulate our physical world and predict its behaviour. In many instances, the accuracy of these simulations is limited by the numerical tools and the mathematical methods. I was curious to understand these methods and also interested in pushing forward these limitations.

When did you first realize that you wanted to pursue a career in applied mathematics research? Which areas initially captured your interest, and how did they influence your research focus?

I professionally started designing structures and modelling damage in concrete, but I realised that I was more interested in developing numerical methods rather than applying them for specific problems. I then opted to fully focus on the numerical aspects, studying the mathematical properties of the differential equations, and compute numerical solutions that can preserve those properties. Somewhat, the experimental and engineering practice required handling too many uncertainties!

Could you share some of your key experiences from your doctoral years at Imperial College? In what ways did this time shape your academic career and influence your research trajectory?

The department of Aeronautics at Imperial College was mainly divided in solids and fluids sections. I joined the former one, which had a very nice combination of experimental work on composites, and a close collaboration with industry. Importantly though, it had built up a strong background on theoretical and computational aspects. The modelling of 3D rotational fields, and its interpolation in space and time, while preserving the integrals of motion was a hot topic at the time, and this opened a completely new view to me, which I was eager to transfer to contact computational mechanics.

Developing quantitative tools for biology and medicine has gained significant traction in recent years. Could you share insights into the most theoretical challenges you've encountered, and what role do mathematical tools play in addressing them?

During my early years I was modelling concrete structures, which had many complexities that I decided to avoid (porosity, moisture, crack, chemical composition, ...), but years later I realised that biological cells and tissues are even more cumbersome. Modelling requires to ignore all unnecessary complexities, and this demands a close collaboration with biologists. In return, mathematical models help to understand and translate into equations the key phenomena, and confirm physical and biological hypotheses, or even to reveal underlying unexpected effects. The rules of collective cell migration, or pointing out the main mechanical contributions during development are some examples.

Your work bridges engineering, mathematics, and biology, forming a highly interdisciplinary field. How do you view the role of interdisciplinary collaboration in advancing research in your field? Could you share examples of how insights from other fields have influenced your approach?

In the last two decades there has been a huge progress in approaching the language of different disciplines. Terms like polarisation, phenotypes/genotypes or morphogens have become a common practice in mathematical modelling, while strains, momenta, tensor, tractions or stresses are also employed by biologists working with modellers. Also the training has been strongly affected: your background and bachelor degree may be less important than your subsequent work and experience. I have witnessed very good work from biologists interested in computational models, or civil engineers that have curated cells and tissues themselves. Being said this, we need to relay on the expertise of other researchers and specialists. Handling all the required technological challenges becomes nowadays unaffordable, but knowing its possibilities is a necessity.

In your view, what are the most promising emerging research directions or open questions in the field of quantitative biology and mathematical modelling of cell formation? Where do you see the field heading in the next few years, and what developments are you most excited about?

As modellers, we can provide tools that can mimic the biological systems and quantify non-measurable variables. I am not only talking about mechanical forces, but also about feedback laws and key variables that determine for instance evolution of cancer progression, cell migration or developmental processes. These tools are though not yet mature enough. For instance, we have been trying to model early embryo development for many decades, but we are still unable to closely predict their shape and functionality beyond a few hundreds of cells. There are biological knowledge gaps, but also mathematical and computational limitations, due to the complexity of processes that are driven by multiple scales and a myriad of cues. Biologists experiment with different organoid types, but in most cases with a try and error approach. The development of robust and high fidelity computational models will become crucial and of great help in this field.

As you step into your new role as a Full Professor, what is your vision for your research in the coming years? What key achievements are you aiming for?

I have become interested in optimal control, and its application to fitting experimental work. This is a recent approach that started circa ten years ago in shape analysis, but that can also help to infer other quantities in cell and tissue mechanics. There are also many interesting open aspects of this theory that span from the symplectic structure of the problem to consistent time integration methods. Also, current technology and microscopy techniques are able to register a massive amount of highly accurate data. Computational models need to catch up with this new inputs, and learn how to incorporate them wisely.

How do you view the role of teaching in your academic career today? How has your approach to teaching evolved alongside your research work?

Clearly, the methodology has changed and will have to keep changing. Currently, students have at hand a vast set of information and tools, more than what they can afford in their time available, which has been in turn reduced. This may become overwhelming and unproductive in some situations. As lecturers, we have the role of guiding their attention, offer effective study methodologies, and implement well focused reference notes. The methods for marking and evaluating their understanding needs also to be adapted.

You have supervised several PhD and master students. How has this mentorship experience influenced your perspective as a researcher, and what have you learned from guiding students in their academic career?

I tend to be overambitious in the objectives. I think though that this is needed, because our work requires a strong enthusiasm. I believe it is important to transfer and communicate this motivation to the students. At the same time, students doubts and questions are of great help for the progress of the research. Each PhD has provided me new open problems to jointly solve. It is therefore important to support a fructiferous dialogue.

You have been involved in the Directory Board of the IMTech since its creation, which are the main weak and strong points of the institute?

IMTech has an enormous potential, not only due to the members that form it, but also thanks to the PhD students and postodcs that are supervised by them. The quality of the research needs to be better visualised, and increase in turn the awareness of its members and interactions among the research groups. I think this is important also for the PhD students themselves, and foster their potential contacts which may become valuable in their future careers. On the other hand, the visibility of the research needs to be materialised in good funding opportunities. The institute needs to make an effort for finding stable funding sources in order to become an attractive pool of activity.

We hope that you will enjoy your Full professorship and that it will turn out to be a most positive time for your academic career. We also wish you good luck!

Many thanks!

▷ Editorial

Toni Guillamon is Full Professor at the Department of Mathematics (**DMAT**), member of **IMTech**, and member of the research group Dynamical Systems , as well as a member of the Centre de Recerca Matemàtica (CRM) .

He received his degree in Mathematics in 1989 and his PhD in Mathematics from Universitat Autònoma de Barcelona (UAB) in 1995 under the supervison of Prof. Armengol Gasull,. He joined UPC in 1994 as Assistant Professor and became Associate Professor in 1997. During the academic year 2000-01 he did a research stay at the Courant Institute (NYU). He has served as secretary of the Departament de Matemàtica Aplicada 1 (1999-2000, 2001-2005), deputy director of the CRM (2011-2015) and vicedirector of coordination of DMAT (2021-2024).

Dr. Guillamon's research primarily focuses on the study of dynamical systems that model biological processes with a particular interest in neuroscience. He has made research on the theory of limit cycles with applications to population dynamics, studying relations between geometric and dynamical aspects in ordinary differential equations, developing tools to study response functions for oscillators and apply them to neuroscience problems, designing methods of estimation of synaptic inputs from neuron's activity and modeling bistable perception and synaptic plasticity in the brain; he has also made other contributions to modeling cancer therapies, neurolinguistics and pharmacology. He has supervised 4 PhD theses, more that 30 master and bachelor theses and participated in about 30 research projects. He is currently the Editor-in-Chief of the Butlletí de la Societat Catalana de Matemàtiques.

NL. ***Congratulations on your recent appointment as Full Professor in Applied Mathematics! Could you share when your interest in mathematics first began, and how it developed throughout your undergraduate studies?***

Thank you! In high school I had a lot of fun solving math problems (among many other things!), but I actually ended up choosing these major quite by accident. It was not until after a couple of years as an undergraduate that I realized what Mathematics represented. In those first courses, I enjoyed discovering the variety of areas of mathematics, and how many fascinating challenges they generated.

At what point did you realize you wanted to pursue research in mathematics? Which areas initially captured your interest?

What captivated me the most, and made me decide to continue in this discipline, were the mathematical models, in particular, the dynamical systems. I found amazing that mathematics offered me a key to understand other fields of knowledge, and the possibility of developing it with experts who had been great teachers was determinant. I think my subconscious had been working to connect my interest in mathematics with my attraction to other fields of knowledge; looking back I see that mathematics has helped me approach other intellectual concerns like language and biology.

In 2000, you were a visiting professor at NYU, where you began collaborating with Professor John Rinzel, a pioneer in computational neuroscience. How significant was this research experience for your career?

t was decisive. For the first time I found myself in a research environment in which models were built in direct relation to the problems posed by experts in other disciplines; in this case, neuroscience. As said above, I had already worked with models, but with mathematical objectives that were not directly connected to central problems in biology. Suddenly, I found myself discussing with experts in neuroscience and, at the same time, maintaining contact with great mathematicians. My mentors were Dave MacLaughlin, a renowned expert in PDEs and then working on the first visual cortex model that was developed at NYU, and John Rinzel, one of the fathers of mathematical neuroscience who had just moved from NIH to NYU. Apart from his deep knowledge on mathematical models in neuroscience, John has an incredible touch to communicate, a special combination of knowledge, empathy and passion, which made me join mathematical neuroscience with enthusiasm. I feel very lucky and privileged to have been introduced to this world by him. Besides, his approach to neuroscience was strongly based on ideas coming from dynamical systems' theory, which of course facilitated my engagement.

What are your key recollections from that period of research?

Being at NYU also brought the opportunity to interact with many other researchers in the area, which at that moment was in a blooming phase, in a strong cooperation between the Courant Institute and the Center for Neural Sciences. I have realized later on that in a short period of time I received an extremely valuable amount of information that has guided my research for many years.

You have worked with researchers from various disciplines, particularly in biology and medicine. How has this interdisciplinary collaboration influenced your work? How has been collaborating with them?

First of all, I must say that I believe that a scientific collaboration is not complete without quality human interaction. In this sense, I feel lucky to have met excellent collaborators along my career. Focusing on the scientific part, a first obstacle in interdisciplinary collaborations is always to find a common language, which is essential to understand exactly what your collaborator needs from you. Second, you cannot constrain yourself to apply very specific mathematical tools: it often happens that you need to learn new maths or, surprisingly, that the problem is mathematically simpler than you expected (not realizing this has made me waste a lot of time!). Even in these simpler cases, it is worth pursuing the problem and keep diving into it because it often leads to non-trivial mathematical challenges; very often, there is an abrupt jump of mathematical difficulty, but this is possibly the main mechanism that has made mathematics evolve along history, although in the last century maths have had also a more endogenous growth. I like playing this role of mathematician "fishing" new mathematical problems while learning from other disciplines.

In your view, what are the main open questions in theoretical neuroscience? How can mathematics contribute to addressing them?

A clear direction is the application of machine learning techniques together with data in order to obtain heuristic models. This approach seems to work against the paradigm of biophysical models in which mathematical neuroscience has grown during decades. In some sense, machine learning, which was initially inspired by the neuronal structure is "paying back" to neuroscience. In my opinion, a great challenge in to reconcile the advances obtained using ML with the power of biophysical models to explain the underlying mechanisms; this poses very interesting mathematical problems related to the explainability and, possibly, a lot of "food for thinking" in areas like dynamical systems, functional analysis, graph theory or topology.

As you step into your new role as a Full Professor, what is your vision for your research in the coming years? What key achievements are you aiming for?

One synthesis exercise that currently appeals to me and that I view as a mid-term goal is conducting an in-depth reflection on the connections between mathematical models from various areas of biology, along with a compilation of the mathematical techniques they employ, which may also involve trying to connect structural and dynamical information.

As an example of my current research, I mention a new collaboration on pharmacology models with researchers from the *Institut de Neurociències de la UAB*. We focus on molecular ligand-receptor binding; a lot of work has been done on equilibrium models, but there is little research on their time dynamics. The models are related to the theory of chemical reaction networks (CRN) and my goal is to explore what types of dynamics (oscillations or even more complex) these specific CRN models can present and to fit them into larger-scale models of brain dynamics.

With your strong track record in teaching, how do you currently view this aspect of your academic career?

I believe that the transmission of knowledge is a fundamental point of our work and, therefore, I take up teaching with great enthusiasm. I try to keep alive the illusion of teaching, of understanding the new generations, despite the fact that the age gap is obviously growing, and training them both on a practical and intellectual level. I cannot envisage the world of research as separate from teaching, despite the individualistic trend in recent decades. From the mathematical side, I consider that we have great challenges ahead: the general preparation of our students is increasingly weak mathematically but at the same time more heterogeneous. In some studies we meet highly motivated and prepared students and, therefore, our task is to guide them to achieve higher goals. On the other hand, in other studies motivation and preparation are scarcer and we have the challenge of getting students to regain some motivation and understand the usefulness of mathematics not only on a practical level but also as a reasoning tool. I like to face both challenges and being active in changing from different subjects within mathematics; in this way, I feel always fresh and ready to learn and communicate. I refuse to believe that a population capable of learning to write cannot also be expected to possess a basic level of scientific, and particularly mathematical, knowledge.

However, given the current state of linguistic proficiency, perhaps I am expecting too much.

You have significant experience in university administration; could you share your reflections on this aspect of academic life?

Yes. I have certainly held positions in the department as well as in a research center. They are two very different experiences. In the research center, the task was much closer to its governance, with much more autonomy but also stronger responsibilities. Despite of the always-present economic restrictions, I could participate in the design of the policies of the center, which was a quite rewarding experience. On the other hand, I have willingly taken on tasks within university departments, mainly driven by a sense of responsibility to give back to the community that supports us as its members. In my opinion, everyone should go through some position to realize the difficulties in making decisions at all levels. In particular, within departments, the lack of autonomy relative to higher decision-making levels is striking. When it comes to human resource planning, university departments often find themselves as the final link in a chain of delayed and occasionally misguided decisions that are already originated in government policies. I am sure that if medium-term decision-making capacity were possible on the part of the departments, it would substantially improve scientific quality. Now, the opportunities to grow come in a disorganized manner that does not allow for optimal talent recruitment.

As a member of IMTech, which strategies would you favor in order to maximize its potential?

It seems to me that the most coherent objective of an institute like IMTech should be cross-fertilization between research groups. However, my impression is that, in general, IMTech members pursue successful and time-consuming research careers and therefore find it difficult to invest time in other research topics; while new challenges are always exciting, they also convey risks that could act as a brake in a demanding environment. Therefore, it is unlikely to expect synergies to emerge without very active intervention. I believe that IMTech's most important task is to engage in a thorough analysis of its researchers' capabilities and interests, proactively forming and convening working groups to explore collaborative opportunities, and identifying calls for proposals where joint projects can be presented.

We hope that you will enjoy your Full professorship and that it will turn out to be a most positive time for your academic career. We also wish you good luck!

Thank you!

▷ Editorial

Víctor Mañosa is Full Professor at the Department of Mathematics (**DMAT**) of the UPC, member of **IMTech**, and member of the research group Dynamical Systems .

He received his degree in Sciences in 1993 and his PhD in Mathematics from Universitat Autònoma de Barcelona (UAB) in 1999 under the supervison of Profs. Armengol Gasull, and Francesc Mañosas. He joined UPC in 1994 with an associate-lecturer position. He has been associate professor and deputy director of communication at DMAT.

Dr. Mañosa's research primarily focuses on the study of dynamical systems in a broad sense. He often laughs together with his colleagues saying that their work turns around periodic orbits, which is their fetish object. Dr. Mañosa particularly likes the study of algebraic aspects of discrete integrable systems, and from time to time he returns to some motivating problems such as the study of traveling waves.

NL. ***Congratulations on your recent appointment as Full Professor in Applied Mathematics! Could you share when your interest in mathematics first began, and how it developed throughout your undergraduate studies?***

Thank you very much! When I was a child, I wanted to be a physicist and to pursue astronomy, which has always been an unfulfilled passion of mine. My interest in Mathematics arose, when I was a teenager, from conversations with my cousin, Alberto Cabada, who is now a Full professor at the University of Santiago de Compostela but who was then a degree student of mathematics, from everything he explained to me while we traveled through the mountains of Galicia in his car. Alberto gave me his book of General Topology and I told myself that I wanted to study it. So when I started my degree, my interest was focused on topology and differential geometry, but I discovered that the qualitative theory of dynamical systems combined these disciplines (and many more!) and was closely aligned with my initial, and never abandoned, interest in Physics. I still find in dynamical systems a fascinating field where I can develop myself.

At what point did you realize you wanted to pursue research in mathematics?

Well... I don't think I realized it (laughs); I think things just happened. When I finished my degree, Prof. Armengol Gasull asked me if I wanted to do my thesis with him, and I simply said yes, because he was an extraordinary teacher! Later, Prof. Francesc Mañosas joined in as co-supervisor... and here we are, more than thirty years later, still colleagues and friends!... teaming up with my dear colleague of so many years, Prof. Anna Cima.

Could you share your experiences during your doctoral years at UAB?

I didn't really experience the typical life of a doctoral student. I spent just one year at the UAB working as an assistant professor, and it was a great year! But, the following year, I joined the UPC at the former Terrassa University School of Industrial Engineering with full teaching dedication, and I poured myself in. I had great colleagues in my department who taught me how to become the professor I am today. I'm, again, grateful for that.

You have also become interested in applied aspects of your research, particularly in areas like modeling and control. Could you share insights into the most theoretical challenges you've

encountered in this field, as well as the practical applications that motivate your research?

I have been fortunate to work on some applied problems, alongside my colleagues from the CodaLab group (my former group), Profs. José Rodellar and Fayçal Ikhouane. Having worked on realistic problems, in addition, has helped me a lot to connect with the interests of my engineering students.

I remember a talk given by V.I. Arnold at the University of Barcelona, where he explained that his mentor, Kolmogorov, always required his undergraduate students to study problems originating in real life. Of course, there were cultural, and even political reasons for this, as we are talking about the former Soviet Union, but there was also something deeply formative about it. These problems are a challenge because nature imposes tremendous constraints that force us to rethink the classical methodologies that we have.

Let me give you an example, naive but real: As a dynamicist, I can study the equations that models the behavior of a structure (a building or a bridge, for instance) and obtain results showing that, under a certain control action, the state variables of my system remain bounded or even asymptotically tend to an equilibrium. But what does that mean when studying an actual structure? Asymptotic behavior has nothing to do here: if I don't guarantee that the positions, velocities, and accelerations of the structure's components remain bounded for all time, and in a very constrained way, the structure will collapse! Furthermore, if we apply an active control, the forces exerted by real actuators must be realistic, you can't do anything. So, a result, even interesting from a theoretical viewpoint, may useless when dealing with the real world, and this is the challenge.

In your view, what are the emerging research directions or open questions in dynamical systems? Where do you see the field heading in the coming years?

A few months ago, we had a meeting of the Dynamical Systems Group at UPC. In it, our younger researchers (young but experienced) took center stage... and it was fantastic to hear them. This generation, that will define the future, is enormously talented, but what struck me the most was their versatility: how they fluidly work on applied problems motivated by biology, physics, or chemistry, but also how they delve into really deep theoretical issues. And that is the key: dynamical systems have always been a liminal field. As long as we are able to incorporate ideas from different areas of mathematics and diverse branches of science, as long as we remain liminal, our discipline will stay alive, and being a source of beautiful results.

As you step into your new role as a Full Professor, what is your vision for your research in the coming years? What key achievements are you aiming for?

Of course, I have research plans. For example, I would like to delve deeper into the arithmetic and geometric aspects of integrable dynamical systems and the nature of their invariant sets. I also have problems that have been open for years, and I would like to return to them with as much intensity as I can. Even so, I've long believed that the most important thing in our work is not the topics we work on, which of course have to interest us, but the people we work with. So my main wish is to keep working with wonderful people from whom I can continue learning, and to whom I, hopefully, can also contribute something. I know that everyday life is very demanding, but as far as possible, I would also like to spend time thinking, organizing and writing down my ideas on the philosophy of science and mathematics. I cannot conceive of doing mathematics without a reflection on its meaning.

With your strong track record in teaching, how do you currently view this aspect of your academic career?

I have been a teacher for more than thirty years. It is the profession I chose to pursue, and I feel great in the classroom, although I get tired much more easily now, something that didn't happen a few years ago! (laughs). I teach to engineering students, and I am grateful to them because they make me rethink mathematics in a way that allows me to explain it meaningfully. In fact, this is my obsession: trying to ensure that what I teach is meaningful in the context of the degree programs I teach. This search for meaning has always kept me a student, and as Professor Nuccio Ordine says in his book 'The Usefulness of the Useless', a teacher is, above all, a tireless student.

You have significant experience in university administration; could you share your reflections on this aspect of academic life?

Well, I don't think I have significant administrative experience. Other colleagues with whom I have had the honor of collaborating have taken on much more demanding responsibilities. In any case, I believe that administrative tasks are a kind work for the community, and that it would be good for all of us to get involved in it in one way or another. In some sense, it helps to cure certain Manichean views because, once again, one must work with reality, with what is possible, and that imposes hard restrictions.

We know that poetry is an important part of your life. You have some published poetry books. How has influenced your vision of the world, and particularly mathematical research?

Poetry, for me, has been a path to self-knowledge. It is a kind of record, but also it is the way I shape my thoughts and give voice to my obsessions. It is, without a doubt, a spiritual activity. And that's the link, because I believe that my approach to science has always been a response to a spiritual need, and that the order I sometimes find in mathematics, that aspiration for beauty, is also a spiritual pursuit.

As a member of IMTech and one of the people who were initially were involved in the creation of the institute, which strategies would you favor in order to maximize its potential?

The UPC is a university with a large territorial area, with numerous campuses, and where different mathematical cultures coexist. In this sense, IMTech can be an instrument to generate a sense of community, and enable cooperation between the different groups, and it will fulfill its purpose to the extent that it has the resources to support the initiatives of these groups. At the recent IMTech meeting, I really enjoyed seeing that master's and doctoral students were taken into account. If they can see the institute as their home, a place where they find support, then things are being done right!

We hope that you will enjoy your Full professorship and that it will turn out to be a most positive time for your academic career. We also wish you good luck!

Thank you very much, really, for your kindness and your time!

▷ Editorial

The laminin-keratin link shields the nucleus from mechanical deformation and signalling by Zanetta Kechagia, Pablo Sáez, Manuel Gómez-González, Brenda Canales, Srivatsava Viswanadha, Martín Zamarbide, Ion Andreu, Thijs Koorman, Amy E. M. Beedle, Alberto Elosegui-Artola, Patrick W. B. Derksen, Xavier Trepat, Marino Arroyo, Pere Roca-Cusachs. Nature Materials volume 22, pages1409–1420 (2023).

The cell skeleton (cytoskeleton) is formed by multiple filaments of different mechanical properties, creating a dense mesh with diverse and complex responses. These cytoskeleton networks interact with the extracellular matrix (ECM) to determine the cell behavior. The paper by Kechagia et al. shows that laminin, a ubiquitous protein in ECM, interacts with the intracellular keratin network to mechanically shield the nucleus of breast cells, and increase its rigidity. These effects provide a mechanism to regulate the sensitivity of cells against external stimuli.

The conclusions of the paper are supported by a model set up by Pablo Sáez and Marino Arroyo, which allows quantifying the role of the actin network, the main component of the cytoskeleton, and myosin motors (other proteins that pull and create the actin flow within the cell) in conjunction with the keratin network. The model is based on a continuum description and considers the mechanical balance of actin and IFs, jointly with transport equations of acto-myosin and keratin network with advection-diffusion effects. The main assumptions of the mathematical model can be found in the Outreach Section in page 13 of this issue.

Importantly, the model incorporates friction and viscosity of actin network, IFs and ECM. The set of coupled non-linear equations is solved by resorting to finite element techniques, which are later used to analyze different scenarios of control cells and mutants. The latter are used to understand important mechanical effects of laminin content in nuclear mechanotransduction.

Quantum error-correcting codes and their geometries, by Simeon Ball (DMAT, **IMTech**)

A *qubit* is a two-state quantum-mechanical system. For example, the intrinsic angular momentum (*spin*) of an electron is such a system. It only takes two values when measured in arbitrary spatial direction, say by measuring the electrons deflection when passing by a magnetic field. The two corresponding spin-states are commonly referred to as as 'spin up" and "spin down" states with respect to that direction. In mathematical terms a qubit is represented by a unit vector in $\mathbb{C}^2$. The spin up and spin down (or any other choice of a pair of physically distinguishable states) are represented by an orthonormal basis $|0\rangle$ and $|1\rangle$. A typical qubit reads

$$|\alpha\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle\,.$$

Let $\overline{z}$ be the complex conjugate of the complex number z. When measured, the qubit is found in state $|0\rangle$ ("spin-up") with probability $\overline{\alpha_0}\alpha_0$ and is found in state $|1\rangle$ ("spin down") with probability $\overline{\alpha_1}\alpha_1$.

The "ket" notation $|\alpha\rangle$ is used for a column vector, whilst the "bra" notation $\langle\alpha|$ is used for a row vector whose coordinates are the complex conjugates of the coordinates of $|\alpha\rangle$. The *inner product* or "bra-ket" on $\mathbb{C}^2$ is defined as

$$\langle\alpha|\,|\beta\rangle = \overline{\alpha_0}\beta_0 + \overline{\alpha_1}\beta_1.$$

The *Pauli matrices*,

$$\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},$$

are unitary linear transformations of $\mathbb{C}^2$ which form a basis for the space of 2×2 matrices.

A system of n qubits is described in the n-fold tensor product space of the one-qubit spaces, the 2^n-dimensional Hilbert space $(\mathbb{C}^2)^{\otimes n} = \mathbb{C}^2 \otimes \cdots \otimes \mathbb{C}^2$ (n times).

The *time evolution* of n qubits is given by unitary operators on $(\mathbb{C}^2)^{\otimes n}$,

$$|\alpha\rangle \mapsto U\,|\alpha\rangle\,.$$

The aim of quantum error-correction is to encode k qubits of information on n qubits in such a way that an error, given by a unitary operator, can be corrected and the correct information restored to the qubits.

The *Pauli group* $\mathcal{P}_n$ of unitary operators is generated by all possible tensor products of the 4 Pauli matrices, together with phases ± 1 or $\pm i$. It is a non-abelian group whose elements either commute or anti-commute ($ab = -ba$). In general, any error can be written as a linear combination of the Pauli operators. Moreover, any linear combination of correctable errors is correctable [8, Theorem 10.2].

A *quantum error-correcting code* is a 2^k-dimensional subspace $\mathcal{Q}$ of $(\mathbb{C}^2)^{\otimes n}$ into which k logical qubits can be encoded such that all errors of a certain type can be corrected. The following theorem from [7], and also [4], details exactly the set of errors that can be corrected by $\mathcal{Q}$.

Theorem 1. *A set of errors $\mathcal{E}$ can be corrected by $\mathcal{Q}$ if and only if for all $|\phi\rangle, |\psi\rangle$ in $\mathcal{Q}$ and errors $E_\mu, E_\nu \in \mathcal{E}$*

$$\langle\phi|\, E_\mu^\dagger E_\nu\, |\psi\rangle = c_{\mu\nu}\, \langle\phi|\, |\psi\rangle\,,$$

for some $c_{\mu\nu} \in \mathbb{C}$.

This condition implies the essential property: orthogonal states in $\mathcal{Q}$ remain orthogonal under the action of errors. The *weight* $\mathrm{wt}(M)$ of an operator M in the Pauli group $\mathcal{P}_n$ is the number of tensor factors which are not equal to the identity matrix. If $\mathcal{E}$ contains all Pauli operators of weight at most t, then the quantum code $\mathcal{Q}$ is a t-error correcting code.

A *qubit stabilizer code* $\mathcal{Q}(S)$ is the joint eigenspace with eigenvalue 1 of the elements of an abelian subgroup S of $\mathcal{P}_n$, i.e.

$$\mathcal{Q}(S) = \{|\psi\rangle \mid M\,|\psi\rangle = |\psi\rangle\,,\ \text{for all } M \in S\}$$

Let $\mathrm{Centraliser}(S)$ denote the set of elements of $\mathcal{P}_n$ that commute with all elements of S, i.e. the centraliser of S in the group $\mathcal{P}_n$. Theorem 1 implies the following theorem [6].

Theorem 2. *$\mathcal{Q}(S)$ can correct all errors in $\mathcal{E} \subset \mathcal{P}_n$ unless there are $E_\mu, E_\nu \in \mathcal{E}$ such that $E_\mu^\dagger E_\nu \in \text{Centraliser}(S) \setminus S$.*

We say $\mathcal{Q}(S)$ is a $[\![n,k,d]\!]$ stabilizer code if $\mathcal{Q}$ is a 2^k-dimensional subspace of $(\mathbb{C}^2)^{\otimes n}$ and the $\text{Centraliser}(S) \setminus S$ contains no Pauli operators of weight less than d.

Let $\mathbb{F}_q$ denote the finite field with q elements. The projective space $\text{PG}(k-1,q)$ is obtained from the vector space $\mathbb{F}_q^k$ by identifying the vectors which are scalar multiples of each other.

Given a subgroup S, generated by $n-k$ commuting elements $M_1, \ldots, M_{n-k}$ of $\mathcal{P}_n$, we obtain a set $\mathcal{X}$ of n lines or possibly points in $\text{PG}(n-k-1,2)$ in the following way. We construct a $(n-k) \times 2n$ matrix $\text{G}(S)$ over $\mathbb{F}_2$, whose j-th is row is obtained from $M_j = c\sigma_1 \otimes \cdots \otimes \sigma_n$, where the $(i, i+n)$ coordinates of the j-th row are given by σ_i, specifically $(0,0)$ if $\sigma_i = \mathbb{1}$, $(0,1)$ if $\sigma_i = Z$, $(1,0)$ if $\sigma_i = X$ and $(1,1)$ if $\sigma_i = Y$,

For each $i \in \{1, \ldots, n\}$, we get a line (or a point) of $\mathcal{X}$ by taking the span of the i-th and $(i+n)$-th column of $\text{G}(S)$.

We define a parameter $d(\mathcal{X})$ as the minimum number of dependent points that can be found on distinct lines of $\mathcal{X}$; not including the dependencies for which there is a hyperplane which both

a) contains all the lines of $\mathcal{X}$ which do not contain the dependent points ,

b) contains all the dependent points.

In the definition of Glynn et al [5], the condition b) does not appear.

The following theorem from [2], which refines the main theorem from [5], describes the geometry of a quantum stabiliser code.

Theorem 3. *The following are equivalent.*

1. *A $[\![n,k,d]\!]$ stabilizer code $\mathcal{Q}(S)$, where S is a subgroup generated by $n-k$ independent commuting elements of $\mathcal{P}_n$ and whose centraliser contains no element of weight one.*

2. *A set of n lines $\mathcal{X}$ spanning $\text{PG}(n-k-1,2)$ with the property that every co-dimension 2 subspace is skew to an even number of the number of lines of $\mathcal{X}$ and for which $d(\mathcal{X}) = d$.*

In the following example e_J denotes the vector in $\mathbb{F}_2^6$ whose j-th coordinate is 1 if and only if $j \in J$.

The $[\![6,0,4]\!]$ code is the sum modulo 2 of 16 planar pencils of lines, see Figure 1. The cyclic structure allows one to check quickly that there are no three collinear points intersecting distinct lines of the six lines of the quantum set of lines. Indeed, the points of weight two obtained by summing two points incident with the quantum lines are cyclic shifts of $26, 36, 46$ and the points of weight three obtained by summing two points incident with the quantum lines are cyclic shifts of 134 and 146. Therefore, the minimum distance of the code is at least 4. The points $e_{126}, e_{34}, e_{16}, e_{234}$ are four dependent points, implying that the minimum distance of the code is 4.

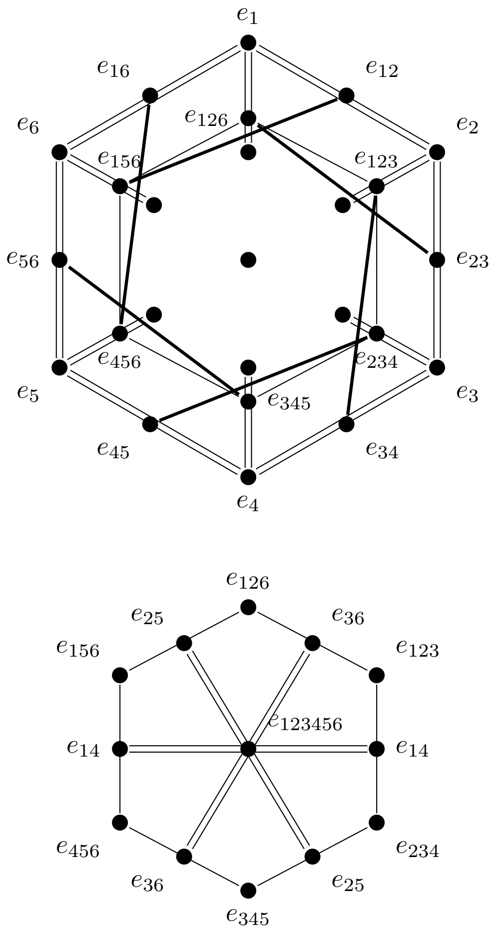

Figure 1: The quantum set of lines (the thicker lines) giving a $[\![6,0,4]\!]$ code.

The geometrical approach to stabiliser codes was used successfully to resolve the long-standing existence question of whether there exists a $[\![13,5,4]\!]$ stabiliser code [3]. The question posed in [2, Research Problem 4] relating to the geometrical object which describes quantum stabiliser codes over non-binary fields has been resolved for fields of even chrcteristic [1]. Defining a quantum set of n symplectic polar spaces of rank h in $\text{PG}(r-1,2)$ as a set $\mathcal{X}$ of n projective $(2h-1)$-spaces spanning $\text{PG}(r-1,2)$ each equipped with a symplectic polarity with the following property: every co-dimension two subspace intersects an even number of the elements of $\mathcal{X}$ in a subspace π for which $\pi^\perp$ is totally isotropic. This geometric object is equivalent to a quantum stabiliser code over $\mathbb{F}_{2^h}$ and resolves [2, Research Problem 4]. Moreover, it was used successfully in the same article [1] to prove the non-existence of the $[\![7,1,4]\!]_4$ and $[\![8,0,5]\!]_4$ stabiliser codes over $\mathbb{F}_4$, another long-standing existence question. We still do not know of any geometric object which gives a useful description of stabiliser codes over fields of odd characteristic.

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PhD highlights

Waleed Mirza defended his PhD thesis *A theoretical and computational study of the active self-organization of nematic patterns in thin cytoskeletal layers and their effect on curvature*, supervised by Marino Arroyo on 16 February 2023 within the UPC doctoral program in Applied Mathematics. Currently he is in the group of Alejandro Torres-Sánchez at the European Molecular Biology Laboratory in Barcelopna, Spain.

D'Arcy Thompson's renowned book "On Growth and Form" (1917) [1] dismisses vitalism, arguing that the growth and forms of living organisms are governed by physical laws and can be mathematically analyzed. Thompson uses analogies between living and nonliving matter, like the similarities in the shapes of soap bubbles, cells, and corals, to argue that physical laws dictate pattern formation without pre-planned design Fig. 2[(a)Top]. This perspective refutes teleology, focusing instead on causality. Over a century later, Thompson's ideas continue to impact the discipline of developmental biology, highlighting the crucial interplay between physics and biology in the self-organizing processes of morphogenesis

Self-organization refers to the spontaneous creation of structured systems through local interactions, characterizing systems far from thermodynamic equilibrium. In contrast to passive systems that increase entropy until equilibrium, driven systems maintain high entropy and energy fluxes by exchanging matter or energy with their environment. This energy input facilitates spatio-temporal organization in various scales, evident in phenomena like ocean circulation, coastline formation [2], to skin patterns on animals [3] Fig. 2[(a)Bottom], and cellular structures.

In the first half of the thesis, we aim to investigate the mechanisms behind the self-organization of active nematic bundles in the actin cytoskeleton [4] Fig. 2[(b)Top]. To this end, we have developed a comprehensive suite of continuum theoretical and computational models tailored to these systems. The initial phase of our thesis focuses on studying these gels on two-dimensional, flat surfaces. Central to our methodology is the development of a systematic modeling approach designed to accurately simulate the dynamics of the actin cytoskeleton. This approach is grounded in Onsager's variational formalism [5], which postulates that the behavior of these systems arises from the interplay of energy release, energy dissipation, and active motion. To enhance our research further, we have introduced an innovative numerical method employing finite element analysis. This method builds upon Onsager's formalism by adding a temporal dimension, ensuring our simulations are not only stable but also in line with the fundamental principles of thermodynamics. Employing this combination of theoretical and numerical frameworks, our study investigates the active self-assembly of nematic patterns, starting from a state where the gel is uniform and quiescent. We use both linearized theory and advanced nonlinear simulations to identify specific conditions required for the formation of nematic bundles from these initial states. Additionally, we examine how various parameters in our theoretical models affect the structure and dynamics of the resulting self-organized nematic patterns Fig. 2[(b) Middle and top]. To reinforce the credibility of our theoretical discoveries, we employ discrete network simulations. This method enables us to corroborate the essential conditions for the self-organization of active nematic patterns, which aligns with the outcomes we previously identified through continuum simulations.

We then explore the functional significance of these self-organized nematic patterns, particularly in cellular processes like wound healing. Our simulations reveal that following a cellular injury, nematic patterns spontaneously align around the wound area. This alignment interacts with the curvature of the wound, playing a crucial role in facilitating the healing process. We validate these findings by comparing our simulation results with in-vitro experiments, finding substantial agreement [6]. Although our primary focus is on sub-cellular patterns within the cytoskeleton, we also venture into the application of our model at a supracellular level. An example of this is the examination of a confluent monolayer of MDCK cells, where similar nematic patterns are observed [7] . Our simulations in this context demonstrate that these self-organized nematic patterns can induce stresses within cell monolayers, leading to the generation of spontaneous hydrodynamic flows [8]. This extension of our model to different scales not only demonstrates its versatility but also provides a deeper understanding of the role of nematic patterns in various biological contexts, from individual cells to complex cellular assemblies.

In the second half of this thesis, our exploration shifts to developing a comprehensive model for self-organized nematic patterns on curved deformable surfaces. Here, we revisit Onsager's nonlinear formalism, a method that balances robustness with simplicity, enabling us to create complex models that are both clear and precise. Our novel model integrates the dynamics of shape changes on curved surfaces, nematic ordering, density variations, and fluid dynamics at the surface level, forming a comprehensive system to investigate their interplay. One of the critical applications of this model is to investigate the self-organization processes of the cytokinetic ring in the actin cytoskeleton, a vital structure in cellular division and migration. The numerical solution of our theoretical model recapitulates the formation of nematic contractile rings, mirroring their significant roles in both cell division Fig.2(c) [9] and migration [10]. Through these simulations, we gain valuable insights into the functionality of these structures, particularly their contribution to the efficiency of morphogenetic processes. This part of our research not only enhances our understanding of nematic structures in the actin cytoskeleton but also underscores the broader significance of these patterns in complex biological systems.

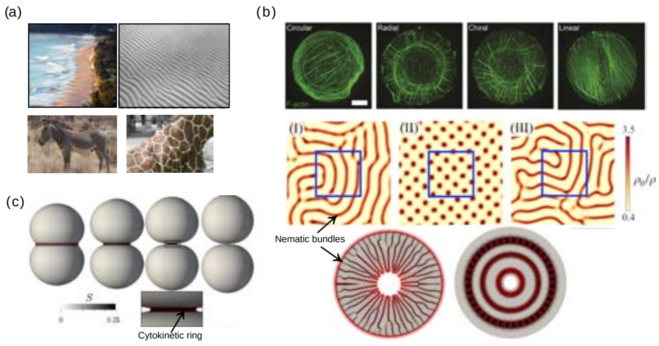

Figure 2: (a) Examples of Self-Organization in Nature (b) Nematic Bundles in Actin Cytoskeleton. Top: Nematic bundles observed in the actin cytoskeleton. Middle: Nematic bundles generated through the numerical solution of the proposed model in a bi-periodic domain. Bottom: Nematic bundles in a circular domain of the actin cytoskeleton. (c) Self-Organized Nematic Bundles Facilitating Cell Division.

Building on our research on nematic models in the actin cytoskeleton, we investigate the application of our model to different surfaces, particularly focusing on vesicles. Vesicles are small, spherical compartments, enclosed by a lipid bilayer, found within cells [11]. They play crucial roles in various cellular processes, including transport, metabolism, and communication between different parts of a cell and with its environment. In this expansion of our study, we apply our nematic model to vesicles, treating them as active, deformable liquid crystalline surfaces. Such surfaces present a unique environment where the irregularities in nematic architecture create out-of-plane deformation. Our theoretical and computational models allow us to meticulously observe and analyze how nematic irregularities in the structured order move and interact with the dynamic shapes of these vesicular surfaces [11].

In this thesis, we achieve a significant advancement in comprehending morphogenesis within biological systems, which echoes and reinforces D'Arcy Thompson's early 20th-century concepts regarding the intricate interplay of biological processes and physical laws in shaping morphogenesis. By developing a novel theoretical and computational framework based on Onsager's variational formalism, we successfully model the interplay of biology and laws of physics. This approach explains the self-organization of prevalent active nematic patterns and their role in morphogenetic events such as cell wound healing, cell division, and cell migration, as well as in other supracellular systems such as patterns in colonies of cells or deformations in vesicles. In summary, these achievements offer profound insights into the fundamental processes of cellular morphology and the universal principles guiding life's development and adaptation.

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Joaquim Brugués defended his PhD thesis *Floer homology for b-symplectic manifolds*, supervised by Eva Miranda and Sonja Hohloch on 20th of March 2024 within the UPC doctoral program in Applied Mathematics and the PhD program (Doctor in Science: Mathematics) at University of Antwerp.

Symplectic geometry is the natural setting for the study of classical mechanics in their full generality. Symplectic manifolds provide an environment for the intersection of the fields of differential geometry, topology and analysis and for results that bridge between them. In that line, Arnold put forward in 1963 a conjecture which proposed that the lower bound to the number of periodic orbits of a Hamiltonian vector field in a compact symplectic manifold is the sum of its Betti numbers. Thus, dynamics are bounded by topology, in a way analogous to that of Morse theory. The general proof of the Arnold conjecture took several decades and inspired key developments in the field. One of the most important of such developments was presented by Floer in the 1980's, taking inspiration from previous work by Conley and Zehnder in variational methods to tackle the Arnold conjecture, and from Gromov's work studying pseudoholomorphic curves. Floer was able to define a homology that combined these approaches to relate the periodic orbits of a Hamiltonian vector field to the homology of the underlying manifold, thus proving the conjecture in a wide range of symplectic manifolds.

Symplectic manifolds can be seen as particular cases of Poisson manifolds, which have a foliated symplectic structure. For instance, the subject of this work is the study of b^m-symplectic manifolds, having a structure that is symplectic almost everywhere but with a singularity in a hypersurface, sometimes called the "singular hypersurface", where the structure is that of a symplectic foliation of maximal rank (namely, of corank 1). In our work we set out to explore possible generalizations of the Arnold conjecture and constructions of Floer-type homologies in b^m-symplectic manifolds. We studied the dynamics of Hamiltonian vector fields in this context, focusing on the presence of periodic orbits, and developed an understanding about how could these be understood as regular symplectic Hamiltonian vector fields under a technique used to study b^m-symplectic topology called *desingularization*.

By means of this desingularization procedure we successfully found lower bounds in the case of b^{2k}-symplectic manifolds. Moreover, when restricting ourselves to b^m-symplectic surfaces we managed to find a strict lower bound. This lower bound is of particular interest because it combines the usual understanding of the homology of a surface with the relative position of the singular hypersurface (in this case, the singular curve) within the surface. This opens an intriguing question about the nature of the Arnold conjecture in the b^m-symplectic setting, in which not only is the topology of the underlying manifold relevant, but also the topology induced by the relative position of the singular hypersurface.

On a slightly different approach, studying pseudoholomorphic curves in b^m-symplectic manifolds, we were able to define a chain complex (and therefore a homology) analogous to that of Floer. However, computing this homology (as in finding an isomorphism to a homology induced by topology) is still an open question.

In this work we also studied the notion of integrable systems in b^m-symplectic manifolds, and more specifically that of b-semitoric systems. We studied the basic properties of these systems and proved that they cannot have singular points in the hypersurface. Then we adapt two important examples of semitoric systems, the coupled spin-oscillator and the coupled angular momenta, into b-semitoric systems by adding a singularity. This provides us with some examples which we are able to study explicitly, and which we hope will provide guidance in some future complete classification of this type of systems.

Selected publications: [1], [2].

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Ashutosh Bijalwan defended his PhD thesis *Numerical optimisation of worm locomotion on frictional substrates*, supervised by José J. Muñoz on 18th July 2024 within the UPC doctoral program in Applied Mathematics. Currently he is in ExxonMobil working as a research engineer in India.

Optimal control problems may be posed as the minimisation of a functional subject to ODE or PDE constraints. The solution of such problems is very relevant from the practical perspective due to the wide type of applications where they may be found: trajectory planning, autonomous vehicles, or shape analysis [1]. The optimality conditions of the state and control variables has been historically controversial, but the prevalent strategy is to resort to the so-called Pontryagin's maximum principle [2]. The resulting set of differential equations for the state, adjoint and control variables share many notable properties with Hamiltonian systems: they have underlying integrals of motion and (pre-)symplectic structures, which interestingly prevail even in the presence of external forces or dissipation effects.

The numerical solution of the optimality conditions is not less challenging, since they combine initial and final value problems, together with algebraic equations. In this thesis we have proposed numerical schemes that preserve as many as the properties of the analytical solution, such as the invariance of the Hamiltonian or the preservation of the flow in the phase-space of state and adjoint variables [3], while garanteeing the stability of the numerical scheme [6].

The thesis adapts the time-integration schemes and solution strategies to problems in continuum elastodynamics and multibody systems [5], and applies them to inverted elastic pendulum or to find the optimal contractility profiles of worm-like bodies [4], as shown in Fig. 3. A salient result of the thesis is that anisotropic friction is necessary for the net motion of the worm, a condition that parallels the celebrated scallop theorem for microswimmers in high Reynolds numbers [7].

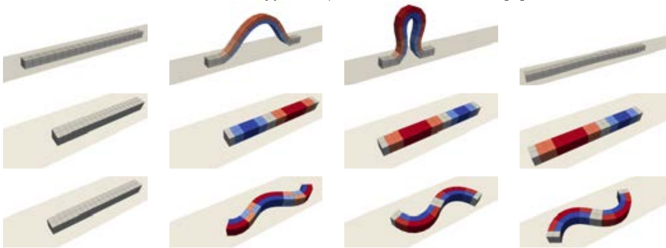

Figure 3: Optimal contractility patterns and corresponding deformations for different worm locomotion strategies. Each row is a different strategy (inching, crowling undulatory), and each column a different time step.

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Outreach

Research that changed the dynamic of fluids turns 100, by Jezabel Curbelo (DMAT, **IMTech**)

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In 1923 Stability of a viscous liquid contained between two rotating cylinders was published, a revolutionary text written by mathematician and physicist Geoffrey Ingram Taylor (1886, London - 1975, Cambridge). The research on which it was based opened new avenues for understanding the patterns that appear in a flow, such as those observed in the movement of the ocean. A century later, the work continues to provide a solid foundation for a wide range of scientific studies and practical applications. For example, it is key to understanding how turbulence develops from a steady flow, i.e., how one goes from an orderly flow to a chaotic and continuously changing one.

The paper is the beginning of the so-called hydrodynamic stability theory. This branch of physics and mathematics tries to understand instabilities in a fluid, small perturbations that cause the flow to deviate from its initial state and evolve into a different configuration. They are formed, for example, by friction between a moving fluid and a solid surface and can amplify over time to cause noticeable changes in the flow, such as the appearances of vortices or eddies.

For centuries, physicians and mathematicians have tried to find a criterion to detect the moment in which instabilities appear, based on the properties of the fluid and the equations and parameters that describe its motion. Lord Rayleigh's theory was a first step in resolving this question: it offered a theoretical model to predict the stability of a fluid without viscosity, i.e., one that does not present resistance to flow. In particular, he showed that the flow is stable as long as a certain condition is satisfied — the square of the angular momentum per unit mass of the fluid increases outwards. This means that if only the inner cylinder rotates, the flow is unstable, while if only the outer cylinder rotates, it is stable.

In 1890, physicist Maurice Couette published his doctoral thesis on the friction between a fluid in movement and a solid surface. In order to measure its viscosity, he designed an experimental device formed by two concentric cylinders — one inside, fixed, and the other outside, rotating — with liquid in between. By rotating the outer cylinder, a flow was generated in the fluid and he was able to quantify the friction produced, i.e., its viscosity. His contribution was so important that Couette's name soon became associated with the flows he studied. At nearly the same time, and independently, Arnulph Mallock, a master of conducting experiments, described the centrifugal instability that occurs when the inner cylinder rotates and the outer cylinder remains stationary.

The next step was to link these experimental observations with Rayleigh's mathematical formalization. This was precisely what Geoffrey Ingram Taylor did, and he realized that, in the case of non-viscous fluids, the experiments agreed with the theory. Taylor, grandson of the celebrated mathematician George Boole, wrote: "It seems doubtful whether we can expect to understand fully the instability of fluid flow without obtaining a mathematical representation of the motion of a fluid in some particular case in which instability can actually be observed." Therefore, to begin to understand — using Rayleigh's mathematical tools — Couette's flow instabilities, it was essential to find a good example to analyze.

Taylor wondered what could be the result of not only one, but both concentric cylinders rotating. He built such a device and with it, using the mathematical tools, predicted the stability of the fluid. To do this, he linearized the Navier-Stokes equations — which describe the behavior of a fluid. That is, he assumed that the pertubations were small enough to neglect part of the terms in the equation, and found solutions of the equations that correspond to the instabilities observed in the experiments.

The equations allowed him to also determine is instability grew — that is to say, if instable flow can give rise to more complex shapes — or decreases with time — that is, whether the flow will remain stable, unchanged. Thus, Taylor was able to theoretically describe the behavior of the fluid, depending on its properties and the rotational velocities of the two cylinders.

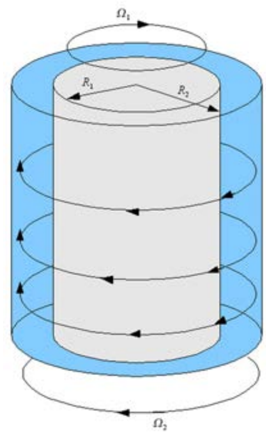

Figure 4: Basic configuration of a Couette-Taylor system

In 1923, he published his work in Philosophical Transactions of the Royal Society. The close agreement between his theoretical and experimental results was unprecedented in the history of fluid mechanics, and as such, the paper is regarded as the first convincing proof of the applicability of mathematical approaches to predict the stability of a fluid.

Taylor's work connected mathematics, physics and engineering and, in the following years, was used in numerous studies related to stability, astrophysical and geophysical flows, nonlinear dynamics or fundamental aspects of turbulence. Since then, this so-called Taylor-Couette flow has been widely studied and has provided a valuable platform for exploring and understanding the basic principles governing the behavior of rotating fluids.

How the transmission of forces between cellular networks explains the expression of transcriptional regulators by Pablo Sáez (LaCàN, **IMTech**)

Cells engage in continuous interactions with the extracellular matrix (ECM). This connection between cells and the ECM is responsible for various cellular and tissue responses, including tissue morphogenesis, repair, and organ development [1]. However, when unregulated, this cell-ECM connection can lead to pathological conditions. Among the factors influencing cell-ECM behavior, the composition and mechanical properties of the ECM are crucial. Central to these interactions are cell membrane receptors called integrins, which act as molecular sensors recognizing specific ECM components [2]. Once bound to the ECM, integrins facilitate intracellular tension transmission, influencing the localization and activity of key transcriptional regulators (genes involved in the regulation of gene expression) like YAP, which is closely linked to cancer initiation [3]. Cells with an overexpression of YAP are highly invasive. Moreover, integrins dynamically respond to changes in ECM mechanical properties and composition, thereby modulating signaling pathways and gene expression.

While extensive research has focused on ECM components like fibronectin and collagen, the precise role of laminin remains unclear. Laminin plays a pivotal role in guiding cellular processes, from maintaining healthy epithelial homeostasis to promoting cancer metastasis. A recent publication led by Jenny Kechagia and Pere Roca-Cusachs from IBEC investigated the role of laminin in cell adhesion [4]. Using MCF10A breast epithelial cells, they observed that cells seeded on laminin showed reduced traction forces and YAP nuclear localization compared to cells on Collagen I and fibronectin substrates. To unravel the mechanisms underlying laminin-mediated effects on nuclear mechanoresponses, they turned their attention to the cell cytoskeleton—a network of protein filaments providing structural support within the cell, and its connections to the ECM. Comprising actin filaments, intermediate filaments (e.g., keratin), and microtubules, the cytoskeleton can link from one side to specific integrins and from the other to the cell nucleus. This chain of intracellular components transmits internal or external forces to the ECM and the nucleus. When it reaches the nucleus, it induces nuclear expression of YAP through several mechanosensitive events. For instance, the keratin network links to the ECM through $\alpha6\beta4$ integrins and to the nucleus through a protein called nesprin-3.

To understand such mechanotransduction chain, and elucidate how force tramission influences YAP expression, they blocked integrin subunits involved in laminin interactions, showing that $\alpha6\beta4$ integrins play a crucial role in modulating the mechanoresponse of the cells. Contrary to expectations, blocking these integrins, which should reduce force transmission to the nucleus, led to increased YAP nuclear localization. However, focal adhesion size or traction forces did not change, which indicated that force transmission neither changed. This raises a puzzling question: if cell adhesion is pivotal for force transmission from the ECM to the nucleus, how can YAP expression change without corresponding changes in traction forces? To further analyze this mechanotransduction chain, the researchers knocked down nesprin-3, a protein connecting the keratin cytoskeleton to the nucleus, reducing YAP nuclear-to-cytoplasmic ratios. This was expected as tension transmission to the nucleus was inhibited. These opposing effects implied that the keratin network alone was not responsible for the changes in YAP expression.

Given the intricate interplay between actin and intermediate filament cytoskeletal networks, it is plausible that the keratin cytoskeleton indirectly affects the nucleus by regulating how actin-mediated force generation reaches it. To explore this hypothesis, Marino Arroyo and I, from LaCàN, developed a computational model of the interaction between these networks and the ECM. To study this cellular system, we described the balance of forces in the actin network as

$$\partial_x \sigma^a = \eta^a v^a + \eta(v^a - v^{IF}) \tag{1}$$

and in the keratin network as

$$\partial_x \sigma^{IF} = \eta^{IF} v^{IF} - \eta(v^a - v^{IF}). \tag{2}$$

The model treated the actomyosin cytoskeleton as an active and viscous gel undergoing turnover, while the keratin cytoskeleton was modeled as a passive viscoelastic gel [5, 6]. Therefore, the constitutive relations for the actin and the keratin network were described as $\sigma^a = \mu^a \partial_x v^a + \zeta\rho$ and $\sigma^{IF} = \mu^{IF}\partial_x v^{IF} - G(\lambda - 1)$, respectively. μ^a and μ^{IF} are the viscosity of the actin and keratin network, respectively. ζ represents the contractility of the myosin motors exerted on the actin filaments, the main source of intracellular force generation, and ρ is the density of the actomyosin network, which was also modeled by a transport equation (see [4]). G is the elastic stiffness of the keratin network and λ is its stretch, which we also model separately. Integrin-mediated adhesions between these networks and the substrate were incorporated with cytoskeletal-substrate friction coefficients. The friction of the actin and keratin network with the ECM is given by η^a and η^{IF}, respectively. Moreover, there is friction between these two networks modeled as $\eta(v^a - v^{IF})$. This coupled system of equations results in the velocity of the actin and keratin network, v^a and v^{IF} respectively. As a result, we can compute the tension on the nucleus and estimate the increase or decrease in YAP expression.

The model predictions revealed that inhibiting the cell-ECM connection through $\alpha6\beta4$ integrins, as experimentally done, results in the actomyosin network dragging the keratin network inward. This manipulation reduced the connection of the keratin network with the ECM and, therefore, its friction coefficient. Consequently, keratin accumulates around the cell nucleus, while actin flow and organization remain largely unaffected. Experimental validation confirmed disrupted keratin organization in mutant integrin-expressing cells compared to controls, while the actin network remained unaffected. However, computation of the traction force on the nucleus showed that these changes in intracellular forces were insufficient to explain the nuclear morphological shape changes measured experimentally, which served as an indicator of tension exerted on them and, consequently, on YAP expression.

The model was successful in recapitulating actin and keratin velocity and distribution but fell short in predicting the actual tension on the nucleus. The missing piece of the puzzle lay in understanding how the mechanical properties of the keratin network change when the connection through $\alpha6\beta4$ integrins is inhibited. Previous publications demonstrated that when the keratin network has lower cross-linking with the ECM, its stiffness reduces. To incorporate this idea, we adjusted the model to make the stiffness of the keratin network, represented by G, proportional to the density of keratin and the adhesion strength, which was also confirmed experimentally. This modification in the model predicted a more pronounced increase in nuclear deformation in mutant cells, as quantified by lower sphericity in the experimental results and, effectively, in an increase in YAP expression.

In summary, through close and interactive collaboration between experimentalists and modelers, we were able to elucidate how cells behave when attached to substrates of different compositions and mechanical properties. We demonstrated how the intracellular transmission of forces between different intracellular structures induces changes in nuclear tension, thereby affecting YAP expression. These findings provide insights into cellular behavior in various contexts where laminin and keratin play crucial roles, such as cancer progression or early developmental stages.

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The circular restricted three-body problem: a modern symplectic viewpoint by Agustín Moreno (Heidelberg University, Germany)

Introduction

The three-body problem is the dynamical system corresponding to three masses under gravitational interaction, as described by Newton's law. It is one of those ancient conundrums that has withstood the ages, maturing like a complex wine. Its study goes back at least to the times of Newton, and its history is tied up with some of the most brilliant scientific figures, like Kepler, Poincaré, Arnold, Moser,... and so many others, who had the courage to try to delve into its well-hidden secrets. The aim of this short note is to describe some of the recent advances in a special case of this problem, made possible through the modern methods of symplectic geometry. In what follows, we will restrict our attention to the *circular* and *restricted* three-body problem (or CR3BP for short), corresponding to the case where one of the masses is negligible, and the other two move in circles around their center of mass. We will focus on the *spatial* problem, where the negligible mass moves in three-space, a opposed to the *planar* problem, where it moves in the plane. Despite the simplifications and approximations, this is still an outstanding open challenge.

The CR3BP is not only interesting from a theoretical point of view, but also from a practical perspective, due to its deep connections to astronomy and space exploration. Namely, the CR3BP is one of the most basic model approximating the motion of a spacecraft under the influence of a Planet–Moon system. This is a modern interpretation: unlike the times of Newton, when space travel was but a wild opium dream, in the current day and age, when mission proposals to remote regions of our expanding Universe are common currency, the CR3BP is one of the preeminent models used for spacecraft trajectory design. In the context of astrodynamics, the CR3BP is then the theoretical starting point supporting the high-fidelity (or *ephemeris*) numerical studies which go into actual mission proposals. While finding trajectories that meet the requirements of an actual mission is a very complicated art, the families of periodic orbits found in the CR3BP, as well as the stable/unstable manifolds of some of them, can be used as building blocks for designing the desired trajectories, and to transfer between them.

We will first discuss some of the theoretical aspects, and then those aspects which are closer to applications. In particular, we will describe a "symplectic toolkit" designed with the needs of trajectory design in mind, the result of a collaboration of the author with NASA engineers. More details can be found in the author's recent book draft [M24] (see also Quanta Magazine's recent article [QM]).

The CR3BP

We consider three bodies: Earth (E), Moon (M) and Satellite (S), with masses m_E, m_M, m_S. One has the following cases and assumptions.

- ❑ **(Restricted case)** $m_S = 0$, i.e. the Satellite is negligible when compared with the *primaries* E and M;
- ❑ **(Circular assumption)** Each primary moves in a circle, centered around the common center of mass of the two (as opposed to general ellipses);
- ❑ **(Planar case)** S moves in the ecliptic plane containing the primaries;
- ❑ **(Spatial case)** The planar assumption is dropped, and S is allowed to move in three-dimensional space.

We denote the *mass ratio* by $\mu = \frac{m_M}{m_E+m_M} \in [0,1]$, and we normalize so that $m_E + m_M = 1$, and so $\mu = m_M$ can be thought of as the mass of the Moon. In rotating coordinates, in which both primaries are at rest, the Hamiltonian describing the problem is actually autonomous. If the positions of Earth and Moon are $E = (\mu, 0, 0)$, $M = (-1+\mu, 0, 0)$, the Hamiltonian is

$$H : \mathbb{R}^3 \backslash \{E, M\} \times \mathbb{R}^3 \to \mathbb{R}$$

$$H(q,p) = \frac{1}{2}\|p\|^2 - \frac{\mu}{\|q-M\|} - \frac{1-\mu}{\|q-E\|} + p_1 q_2 - p_2 q_1.$$

By preservation of energy, this means that it is a preserved quantity of the Hamiltonian motion. The planar problem is the subset $\{p_3 = q_3 = 0\}$, which is clearly invariant under the

Hamiltonian dynamics. The two parameters in the problem are the *Jacobi constant* c (the energy value), and μ.

As computed by Euler and Lagrange, there are precisely five critical points of H, called the *Lagrangian points* $L_i = L_i(\mu)$, $i = 1, \dots, 5$, ordered so that $H(L_1) < H(L_2) < H(L_3) < H(L_4) = H(L_5)$. The *low-energy* range corresponds to $c < H(L_1)$ (or slightly above).

For $c \in \mathbb{R}$, consider the energy hypersurface $\Sigma_c = H^{-1}(c)$. If

$$\pi : \mathbb{R}^3 \backslash \{E, M\} \times \mathbb{R}^3 \to \mathbb{R}^3 \backslash \{E, M\},\ \pi(q,p) = q,$$

is the projection onto the position coordinate, we define the *Hill's region* of energy c as

$$\mathcal{K}_c = \pi(\Sigma_c) \in \mathbb{R}^3 \backslash \{E, M\}.$$

This is the region in space where the Satellite of energy c is allowed to move. If $c < H(L_1)$, then $\mathcal{K}_c$ has three connected components: a bounded one around the Earth, another bounded one around the Moon, and an unbounded one. Denote the first two components by $\mathcal{K}_c^E$ and $\mathcal{K}_c^M$, as well as $\Sigma_c^E = \pi^{-1}(\mathcal{K}_c^E) \cap \Sigma_c$, $\Sigma_c^M = \pi^{-1}(\mathcal{K}_c^M) \cap \Sigma_c$. As c crosses $H(l_1)$, $\mathcal{K}_c^E$ and $\mathcal{K}_c^M$ get glued to each other into a new connected component $\mathcal{K}_c^{E,M}$, topologically their connected sum. Then, the Satellite in principle has enough energy to transfer between Earth and Moon. See Figure 5.

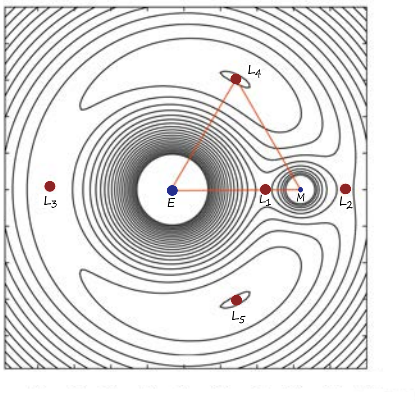

Figure 5: The Hill regions and the Lagrange points for the planar problem.

Theoretical aspects

The Poincaré–Birkhoff theorem, and the planar CR3BP

Poincaré [P87, P12] reduced the problem of finding periodic orbits in the planar CR3BP to:

(1) Finding a global surface of section for the dynamics;

(2) Proving a fixed point theorem for the resulting first return map.

This is the setting for the celebrated Poincaré–Birkhoff theorem.

Collision regularization

The 5-dimensional energy hypersurfaces in the spatial CR3BP are non-compact, due to collisions of the massless body S with one of the primaries. Two body collisions can be regularized via Moser's recipe. The bounded components Σ_c^E and Σ_c^M (for $c < H(L_1)$), as well as $\Sigma_c^{E,M}$ (for $c \in (H(L_1), H(L_1) + \epsilon)$), are thus compactified to compact manifolds $\overline{\Sigma}_c^E \cong \overline{\Sigma}_c^M \cong S^3 \times S^2$, and $\overline{\Sigma}_c^{E,M} \cong S^2 \times S^3 \# S^2 \times S^3$. In the planar problem, we obtain copies of $\mathbb{R}P^3$ and $\mathbb{R}P^3 \# \mathbb{R}P^3$. We use the notation $\overline{\Sigma}_{P,c}^E$, $\overline{\Sigma}_{P,c}^M$ and $\overline{\Sigma}_{P,c}^{E,M}$ for the corresponding planar regularized energy level sets.

The advent of contact geometry in the CR3BP

It was only recently that the modern techniques from contact and symplectic geometry (holomorphic curves, Floer theory,...) have been made to bear on the CR3BP. This is due to the following result.

Theorem 1 ([AFvKP12] (planar problem), [CJK18] (spatial problem))**.** *If $c < H(L_1)$, the regularized hypersurfaces $\overline{\Sigma}_c^E, \overline{\Sigma}_c^M, \overline{\Sigma}_{P,c}^E, \overline{\Sigma}_{P,c}^M$ carry contact structures. The same holds for $\overline{\Sigma}_c^{E,M}, \overline{\Sigma}_{P,c}^{E,M}$, if $c \in (H(L_1), H(L_1) + \epsilon)$ for sufficiently small $\epsilon > 0$.*

Open book decompositions

We have the following fundamental notion from smooth topology.

Definition 1 (Open book decomposition)**.** *Let M be a closed manifold. A (concrete) open book decomposition on M is a fibration $\pi : M \backslash B \to S^1$, where $B \subset M$ is a closed, codimension-2 submanifold with trivial normal bundle. We further assume that $\pi(b, r, \theta) = \theta$ along some collar neighbourhood $B \times \mathbb{D}^2 \subset M$, where (r, θ) are polar coordinates on the disk factor.*

B is called the *binding*, and the closure of the fibers $P = P_\theta = \overline{\pi^{-1}(\theta)}$ are called the *pages*, which satisfy $\partial P_\theta = B$ for every θ. We usually denote a concrete open book by the pair (π, B), and also use the abstract notation $M = \mathbf{OB}(P, \varphi)$, where φ is a diffeomorphism $\varphi : P \to P$ with $\varphi = id$ near B (the *monodromy*). See Figure 6.

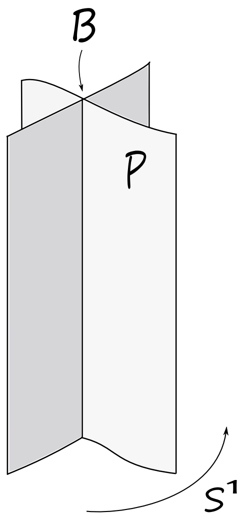

Figure 6: A neighbourhood of the binding look precisely like the pages of an open book, whose front cover has been glued to its back cover via some gluing map (the monodromy).

If M is oriented and endowed with an open book decomposition, then the natural orientation on the circle induces an orientation on the pages, which in turn induce the boundary orientation on the binding.

Definition 2 (Giroux)**.** *Let (M, ξ) be an oriented contact manifold, and (π, B) an open book decomposition on M. Then ξ is* supported *by the open book if one can find a positive contact form α for ξ (called a* Giroux form*) such that:*

1. *$\alpha_B := \alpha|_B$ is a positive contact form for B;*
2. *$d\alpha|_P$ is a positive symplectic form on the interior of every page P.*

Here, a positive *contact form is a contact form α on M^{2n-1} such that the orientation induced by the volume form $\alpha \wedge d\alpha^{n-1}$ coincides with the given orientation on M.*

We say that the open book is *adapted to the dynamics* of α if $(1)'$ and $(2)'$ hold. We have the following foundational result.

Theorem 2 (Giroux [Gir02])**.** *Every open book decomposition supports a unique isotopy class of contact structures. Any contact structure admits a supporting open book decomposition.*

Here, two contact structures are isotopic if they can be joined by a smooth path ξ_t of contact structures. By *Gray's stability*, isotopic contact structures are *contactomorphic*, i.e. there exists a diffeomorphism which carries one to the other.

Remark 1. *In fact, Giroux's result is stronger, as there is in fact a correspondence between contact structures up to isotopy and open books up to a notion of positive stabilization. Giroux proved this in dimension 3, and was recently established in higher dimensions by Breen–Honda–Huang* [BHH].

We usually write $(M, \xi) = \mathbf{OB}(P, \varphi)$ to denote that the contact structure ξ is supported by the open book (P, φ).

Giroux correspondence reduces the topological study of contact manifolds to the topological study of open books. *However,* this result holds only when the (isotopy class of the) contact *structure* is fixed, and the contact form (and hence the dynamics) is auxiliary; Giroux's result is *not* dynamical, but rather topological/geometrical. But this will serve as motivation for what comes next.

Open books in the CR3BP

Let $\overline{\Sigma}_c$ denote either $\overline{\Sigma}_c^E$, $\overline{\Sigma}_c^M$ or $\overline{\Sigma}_c^{E,M}$ (for the spatial problem). The following result generalizes the approach of Poincaré in the planar problem (i.e. Step (I)) to the *spatial* problem. Combining Theorem 1 with Giroux correspondence, we know there exist supporting open book decompositions on $\overline{\Sigma}_c$ for such c in the low energy range. However, as we emphasized already, this correspondence does not give adapted open books whenever the dynamics is fixed. The content of the following result is that the *given* dynamics of the spatial CR3BP in the low-energy range, and near the primaries, is given by a contact form which is a Giroux form for some concrete open book.

Theorem 3 (Moreno–van Koert [MvK20a])**.** *For any $\mu \in [0, 1]$, if c lies in the low-energy range, $\overline{\Sigma}_c$ admits a supporting open book decomposition for energies $c < H(L_1)$ that is adapted to the dynamics. Furthermore, if $\mu < 1$, then there is $\epsilon > 0$ such that the same holds for $c \in (H(L_1), H(L_1) + \epsilon)$. The open books have the following abstract form:*

$$\overline{\Sigma}_c \cong \begin{cases} (S^*S^3, \xi_{std}) = \boldsymbol{OB}(\mathbb{D}^*S^2, \tau^2), \\ \quad \text{if } c < H(L_1) \\ (S^*S^3, \xi_{std}) \# (S^*S^3, \xi_{std}) = \boldsymbol{OB}(\mathbb{D}^*S^2 \natural \mathbb{D}^*S^2, \tau_1^2 \circ \tau_2^2), \\ \quad \text{if } c \in (H(L_1), H(L_1) + \epsilon), \mu < 1. \end{cases}$$

In all cases, the binding is the planar problem

$$B = \overline{\Sigma}_{P,c} = \begin{cases} (S^*S^2, \xi_{std}), \\ \quad \text{if } c < H(L_1) \\ (S^*S^2, \xi_{std}) \# (S^*S^2, \xi_{std}), \\ \quad \text{if } c \in (H(L_1), H(L_1) + \epsilon), \mu < 1. \end{cases}$$

Here, $\mathbb{D}^*S^2$ is the unit cotangent bundle of the 2-sphere, τ is the positive Dehn–Seidel twist, and $\mathbb{D}^*S^2 \natural \mathbb{D}^*S^2$ denotes the boundary connected sum of two copies of $\mathbb{D}^*S^2$. The monodromy of the second open book is the composition of the square of the positive Dehn–Seidel twists along both zero sections (they commute).

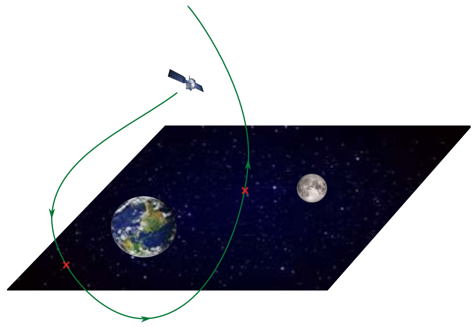

Figure 7: Theorem 3 admits a physical interpretation: away from collisions, the orbits of the negligible mass point intersect the plane containing the primaries transversely. The "pages" $\{q_3 = 0, p_3 > 0\}$, $\{q_3 = 0, p_3 < 0\}$ of the "physical" open book $(q, p) \longmapsto \frac{q_3 + ip_3}{\|q_3 + ip_3\|} \in S^1$, are global hypersurfaces of section outside of the collision locus.

Practical aspects

A given Hamiltonian system usually depends on parameters (e.g. energy or mass parameters), which one may vary. Under such deformation, periodic orbits may undergo *bifurcation*, a mechanism by which new families of periodic trajectories arise. The way different families can connect to each other is encoded in the topology of a *bifurcation graph*. The aim of this Section is then to introduce a "symplectic toolkit", extracted from the modern methods of symplectic geometry, and designed to systematically map out how different orbit families merge together.

Symplectic data analysis

The "symplectic toolkit" consists of the following elements:

(I) **Floer numbers:** Integers which stay invariant before and after a bifurcation, and so can help predict the existence of orbits.

(2) **The B-signs** [FM]: a $\pm$ sign associated to each elliptic or hyperbolic orbits, which helps predict bifurcations, and generalizes classical Krein theory [Kre1, Kre2].

(3) **Global topological methods:** the *GIT-sequence* [FM], a sequence of spaces whose global topology encodes bifurcations, and refines Broucke's stability diagram [Bro69] by adding the B-signs.

(4) **CZ-index:** [CZ84, RS93] a winding number associated to non-degenerate orbits, extracted from the topology of the symplectic group. It can be used to determine which families connect to which.

Numerical work

We now describe some numerical work where we put the symplectic toolkit into practical use. This is based on the article [AFvKKM].

We consider the *Jupiter-Europa system* (JE), which corresponds to a CR3BP with $\mu = 2.5266448850435e^{-05}$, and the *Saturn-Enceladus system* (SE), $\mu = 1.9002485658670e^{-07}$. These two systems are of tremendous current interest for space agencies as NASA, as they may have conditions suitable for life. Starting from the Hill 3BP, we deform to JE/SE. One symmetry is broken, and families behave more generically. In what follows, we use $\Gamma = -2c$.

Planar direct/prograde orbits

The planar pitchfork bifurcation described by Hénon [He69] in the Hill 3BP (concerning planar orbit families f, g, g') becomes a generic broken bifurcation in the planar JE CR3BP, see Figure 8.

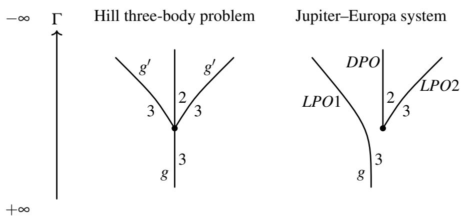

Figure 8: Bifurcations of planar direct/prograde orbits with corresponding CZ-index.

Spatial bifurcation graphs between planar prograde and retrograde orbits

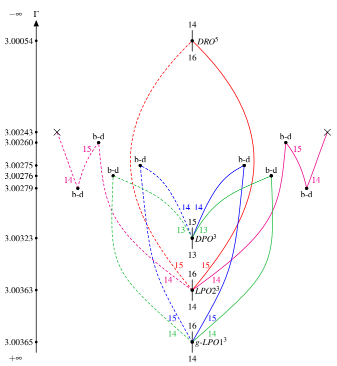

Figure 9: Bifurcation graph for JE, between g-$LPO1^3$, DPO^3, $LPO2^3$, and DRO^5.

A bifurcation graph relating third covers of the direct orbits g, g', and fifth covers of planar retrograde orbits f, connected via spatial families, was obtained by Aydin [Ay22]. We deformed it to JE in Figure 9.

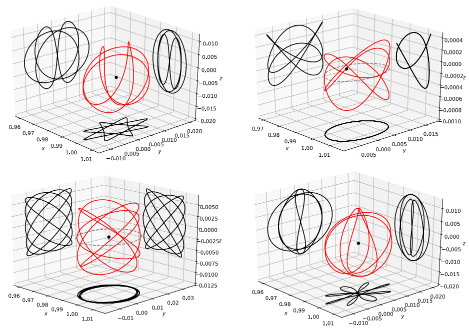

Figure 10: Prograde to retrograde spatial connection, red CZ-index 15 in Figure 9.

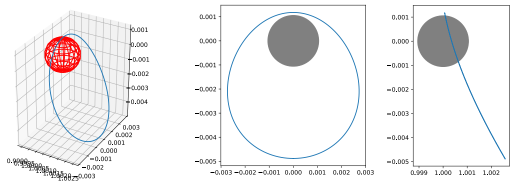

Figure 11: Halo-polar orbit ($\Gamma = 3.000034709155895$) with an altitude of 29 km. The CZ-index has just jumped to 4, and is doubly elliptic.

Halo orbits in SE

We consider Halo orbits coming out of the Lagrange point L_2 in SE. This family appears also in NASA's technical report on the Enceladus Orbilander [MGKM], and is meant to be used to visit the poles in future missions. The corresponding family for the Earth–Moon system is currently very popular, as it will be where NASA's Gateway Space Station will be parked. The most interesting part of the family in SE occurs just after CZ-index jumps from 3 to 4, where orbits are stable and close to the water plumes.

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Chronicles

Michel Talagrand receives the Able Prize 2024,

by Gábor Lugosi (Department of Economics, Univesitat Pompeu Fabra)

The 2024 Abel prize was awarded to Michel Talagrand, –citing the laudation–"for his groundbreaking contributions to probability theory and functional analysis, with outstanding applications in mathematical physics and statistics."

Talagrand's contributions mostly belong to probability theory but they have had an enormous impact in a variety of fields such as asymptotic geometric analysis, mathematical statistics, statistical learning theory, and combinatorics. He is possibly the probabilist whose work has had the greatest impact across and beyond mathematics.

Among his high-impact contributions, one has to mention his ground-breaking work on the theory of measure concentration. Over the 1980's and 1990's he singlehandedly revolutionized the area by deeply understanding the connection between discrete isoperimetric problems and concentration inequalities of general functions of independent random variables.

In his remarkable paper "A new look at independence" (1996, Annals of Probability), he summarizes the concentration of measure phenomenon as

"A random variable that depends (in a "smooth" way) on the influence of many independent variables (but not too much on any of them) is essentially constant."

Talagrand developed a completely new inductive technique to quantify such a statement in the form of inequalities. These concentration inequalities had an immediate impact on a wide variety of areas that are still being exploited daily. Talagrand's approach and way of thinking about the problem was absolutely original.

Another key contribution of Talagrand is the theory of majorizing measures and generic chaining. Starting with his 1987 Acta Mathematica paper in which he characterized the boundedness of Gaussian processes, he has developed a complete theory, culminating in his 2014 monograph "Upper and lower bounds for stochastic processes: modern methods and classical problems". This theory has countless applications not only in empirical process theory but also in harmonic analysis, the geometry of Banach spaces, and in mathematical statistics (e.g., in sparse recovery).

The diversity and depth of his work are absolutely remarkable. He has made important contributions to statistical physics, for example, by proving the Parisi formula for spin glasses that had been a major open problem in mathematical physics. His monumental two-volume monograph "Mean Field Models for Spin Glasses" summarizes his work in this area.

He was the first to understand the deep connection between concentration, isoperimetry, and threshold phenomena, making many key contributions to the foundations of the field of the analysis of Boolean functions that has become a booming area in theoretical computer science.

He made key contributions to the theory of transportation of measure that have deep implications to concentration inequalities.

His book with Michel Ledoux on the Probability of Banach spaces has had a lasting impact on the geometry of Banach spaces, empirical process theory and even statistical learning theory, the most standard mathematical theory of machine learning.

Talagrand is an original thinker and an independent mind, a most deserving recipient of the Abel prize, the most prestigious recognition of a career in mathematics.

The next five items of his section are devoted to the 9th European Congress of Mathematics, that took place in Sevilla in July 15-19, 2024. We review the work of the ECM prize winners Maria Colombo, Tom Hutchcroft, Frederick Manners, Richard Montgomery, and Danylo Radchenko. The remaining awardees were Cristiana de Filippis, Jessica Fintzen, Nina Holden, Jacek Jendrej, and Adam M. Kanigowski.

Maria Colombo receives the ECM Prize 2024,

by Michelle Dolce (EPFL) and by Xavier Fernández Real (EPFL)

Last July, during the events of the 9th European Congress of Mathematics celebrated in Seville, the European Mathematical Society awarded its most prestigious prize for young mathematicians, the EMS Prize. One of the awardees was Maria Colombo, from the Swiss Federal Institute of Technology Lausanne (EPFL), a prize that joins her collection, which in the recent years includes the Bartolozzi Prize (2019), the Peter Lax award (2022), the Collatz Prize (2023), and the also very recent Stampacchia Medal (2024).

Colombo was awarded the EMS Prize *"for breakthrough results in fluid dynamics, optimal transport, and kinetic theory, and for her impact on analysis more broadly"*. We believe this to be a particularly fitting description of the work she has been doing in recent years: Colombo is one of the rare mathematicians whose research does not focus on a single topic but spans the entire spectrum of Analysis. As such, it would be unfair to limit her merits to just one particular subject. In addition to the aforementioned areas, she has also made significant contributions to the calculus of variations, Schauder theory and elliptic equations, free boundary problems, conservation laws, and even machine learning. It is for this reason that it is particularly difficult to summarize her work in a single note, and some choices have to be made when attempting to capture the breadth of her achievements.

Among Colombo's spectacular contributions to the field of Partial Differential Equations (PDEs), those that arise in the description of fluids (such as the modeling of whirlpools in water) shine with their own light. She is particularly known for her work on the nonuniqueness of solutions to the transport, Euler, and Navier-Stokes equations.

This is a fundamental theoretical question crucial for validating the use of models described by these equations, specifically regarding the predictability of certain events. For example, if a particle is placed in a given flow of water at a fixed position, will it always end up in the same location? It is also a possible way to understand some features of the turbulence of fluids. Indeed, it is a common understanding that fluids can behave in an unpredictable, chaotic way, and that even a small perturbation of the status of a fluid can be amplified and have important effects later in time. However, the quantitative and theoretical description of turbulence is a major open problem in the mathematical community. Important mathematicians and physicists, such as Kolmogorov and the Nobel Prize winner Onsager, developed a phenomenological theory with certain precise mathematical questions, connecting turbulence to the mathematically described nonunique, irregular solutions of the fluid dynamics equations.

Colombo (and her collaborators) provided a negative answer to the uniqueness question across different models using a variety of mathematical tools and techniques: a feat that is even more impressive considering the diversity of approaches involved.

She introduced new and profound ideas in convex integration, a technique used to construct highly complex objects through iterative schemes. For instance, she constructed 'wild' solutions to the 3D Navier-Stokes equations by introducing the concept of intermittent jets, [4]. Moreover, in a recent preprint [2], she solved the long-standing open problem of nonuniqueness of solutions to the 2D Euler equations with integrable vorticity, significantly improving the best result available to date [3] (which was also her contribution).

Recently, the problem of nonuniqueness has been elegantly linked to spectral stability properties, and Colombo achieved remarkable results here as well for the 3D Navier-Stokes equations [1], in one of her recent ground-breaking contributions. These equations are used to describe the evolution of a (incompressible) fluid via a velocity field, which points in the direction that each particle is locally moving into. Surprisingly, despite its many applications, mathematicians have not yet been able to prove a well-posedness and smoothness theory for these equations; in fact, the Navier-Stokes existence and smoothness problem is one of the MIllenium Prize Problems for which the Clay Institute offers one million dollars upon finding a solution.

In this direction, in a seminal work from 1934 [6], Leray demonstrated the existence of global weak solutions to the 3D Navier-Stokes equation in the whole space, which combined with the results of Hopf in domains 20 years later, [5], gave rise to what nowadays are known as Leray-Hopf solutions, satisfying certain desirable properties, and for which the community wondered whether they constituted an existence and uniqueness class of solutions for the Navier-Stokes equations. It had been clear for some years that these solutions seemed to present non-uniqueness issues, and Colombo showed in [1] that it is indeed the case for the forced Navier-Stokes equations (with a force term under reasonable integrability properties and within the Leray-Hopf class). The new nonunique solutions are much smoother and emerge from a standard spectral instability. This methodological difference also leads to profoundly different physical properties of the solutions, offering a variety of possible answers to the same foundational question. Quanta Magazine referred to this discovery as "Mathematicians Coax Fluid Equations Into Nonphysical Solutions" in their **article** about this paper.

Colombo's work represents a significant leap forward in our understanding of fluid dynamics in particular and mathematical analysis more broadly. Her ability to tackle some of the most challenging problems in PDEs with innovative methods and profound insights places her as one of the leading mathematicians of her generation. The recognition she has received is indicative not only to her groundbreaking contributions but also to the broad impact her research has had across multiple fields.

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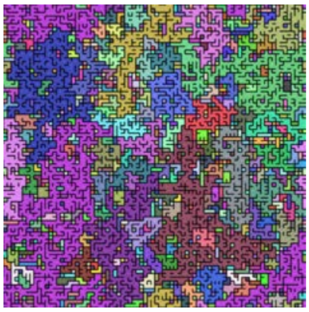
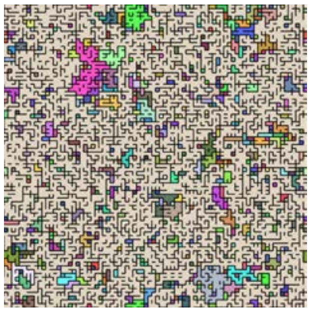
Tom Hutchcroft: an EMS prize for important progress on percolation theory, by Juanjo Rué (DMAT, **IMTech**)

As it is stated in Hutchcroft's laudatio, this EMS Prize has been awarded *"for his revolutionary contributions to probability theory and geometric group theory, in particular to percolation theory on general graphs, using tools from geometry, operator theory, group theory and functional analysis."*. In this short text we will describe possibly one of his most important achievements: his resolution (joint with his PhD student Philip Easo) of the so-called *Schramm locality conjecture* (see [2]). This will show the taste and the kind of results in which Hutchcroft has made decisive contributions.

Percolation theory is a fascinating and deep topic in the interplay of probability theory and mathematical physics, with very important results in the last recent years due to different authors (just to cite three of them, the Field medallist Wendelin Werner, Stanislav Smirnov and Hugo Dominil-Copin had made decisive contributions to this area). There are different models (most of them inspired by physics) to study this notion. Just to get an idea we will consider the following model: consider an infinite connected graph G where all vertices have finite degree (namely, *locally finite*). We also assume that our graph is transitive, which means that for any pair of vertices v_1 and v_2 there is an automorphism of the graph that maps v_1 to v_2 (for instance, the rectangular grid defines a transitive graph where all vertices have degree 4). A good example of such graphs are Cayley graphs, which are defined from groups and have nice symmetry properties. Now delete or retain each edge independently with a certain probability p or $1-p$. Which is now the geometry of the subgraph that we obtain, or more precisely, which is the structure of its connected components? Are they all finite, or some of them is infinite?

Much of the interest in these type of models arises from the fact that there exists a *phase transition*. In our case study, there is a critical probability $p_c(G)$ from which the general picture of the graph obtained dramatically depends on wether p is smaller or bigger than p_c. In particular, the value of p_c rules the existence or not of an infinite connected component (almost surely). For instance, again in the rectangular grid we can see a dramatic change of behaviour when passing the value $p_c = 1/2$. In fact, a celebrated result by Kesten [3] states that $p_c = 1/2$ is indeed the probability in which there is a sudden change, and that at $p_c = 1/2$ there is not an infinite cluster.





Two samples of the percolation model on the rectangular grid for $p = 0.46$ and $p = 0.51$. On the sample on the right, there is an infinite cluster.

The interesting features of percolation on an infinite transitive graph, especially around the critical point, are expected to be *universal*. This means they depend only on the graph's overall shape, not its local details, for instance finite considerations. In the opposite direction, Oded Schramm (who introduced novel ideas coming from conformal field theory into probability theory in a way that was advanced for his time) conjectured in 2008 (see [1]) that the value of the critical probability $p_c(G)$ should be entirely determined by the local (microscopic) geometry of the graph, subject to the global constraint that $p_c(G) < 1$. Some cases have been known to be true but only last year, in a 87-page paper Huchcroft and Easo fully solved this conjecture.

The solution gives a better insight into what happens above the percolation threshold. Huchcroft techniques open the door to study what happens exactly at the threshold for most graphs: is there an infinite cluster at $p_c(G)$. Even in the three-dimensional rectangular grid this is still an open question.

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Frederik Manners receives the ECM Prize 2024, by Pablo Candela (ICMAT)

Frederick Manners has been awarded one of the European Mathematical Society prizes in recognition of his outstanding contributions in arithmetic combinatorics and related areas. Manners, who is currently an associate professor at the University of California, San Diego, has been particularly praised for his work in the area known as higher-order Fourier analysis. We will focus here on two of his main contributions.

A first highlight is the quantitatively effective proof that Manners gave in 2018 of the inverse theorem for Gowers norms (or U^d norms) on cyclic groups $\mathbb{Z}_N$ (the so-called "integer setting") [5]. These norms were introduced by Gowers in his famous work on Szemerédi's theorem in 2000 [1]. Since then, they have become central tools in arithmetic combinatorics, especially useful in relation to detecting and counting various kinds of linear configurations in subsets of abelian groups. These norms have also led to a generalization of classical Fourier analysis. In the classical theory, a function on a compact abelian group is decomposed into fundamental harmonics (the Fourier characters) whose underlying structure is based on the circle group. The smallest of the Gowers norms, the U^2-norm, is closely related to this classical theory. However, the U^d norms of higher order (i.e. for $d > 2$) cannot be analyzed in terms of classical Fourier characters, requiring instead more subtle underlying objects, such as nilmanifolds. This motivated the above-mentioned generalization, a theory known as higher-order Fourier analysis. This theory includes as a central result the so-called Inverse Theorem for the Gowers norms, proved first in the integer setting by Green, Tao and Ziegler in 2010 [4]. This theorem asserts that if a function f of modulus at most 1 on a large cyclic group $\mathbb{Z}_N$ has non-trivially large Gowers U^{d+1}-norm, then f has large inner product with a d-step nilsequence of bounded complexity, a function which plays the role of a Fourier character relative to this norm, and which is defined using a d-step nilmanifold (instead of the circle group for classical characters). The initial proofs of this inverse theorem did not provide effective bounds for the complexity of the nilsequence or the magnitude of the inner product in the conclusion for general order d (good bounds were known in special cases, notably for the U^3-norm due to Green and Tao). It was an outstanding achievement of Manners to provide the first reasonable bounds for these quantities for general order d.

The second highlight is the outstanding proof, given in 2023 by Manners jointly with Gowers, Green, and Tao, of Marton's conjecture (also known as Polynomial Freiman–Ruzsa conjecture) in additive combinatorics, in the case of vector spaces over finite fields [2]. This conjecture, well-known in this area at least since the 1990s, can be stated as follows: suppose that a subset A of $\mathbb{F}_2^n$ has a small sumset in the sense that $|A+A| \leq K|A|$ for some constant $K>1$. Then A can be covered by at most $2K^C$ cosets of some subgroup $H \leq \mathbb{F}_2 n$ of cardinality at most $|A|$ (for some absolute constant C). The conjecture was widely known to be of central importance in the field, with many equivalent reformulations, and strong connections with other principal results in this area. Notably, an equivalence had been established by Green and Tao in 2010 [3] between this conjecture and an effective version of the inverse theorem for the U^3 norm with polynomial bounds. The proof of Marton's conjecture given by Manners and his coauthors is outstanding in many aspects, especially for the use it makes of entropy tools.

These examples are only a few of the important mathematical contributions of Manners to the development of the highly conceptual and technical fields of arithmetic combinatorics and higher-order Fourier analysis.

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Richard Montgomery receives the ECM Prize 2024, by Tássio Naia (CRM)

Richard Montgomery was one of the 10 recipients of the prestigious European Mathematical Society (EMS) prize at the 9th European Congress of Mathematics, in Sevilla. This prize is awarded every four years by the EMS to mathematicians under the age of 35, in recognition of exceptional contributions to mathematics. His award announcement credits him for *his solution of the Ringel tree packing conjecture, development of distributive absorption techniques with applications to graph embedding problems, and resolution of several classical conjectures of Erdős and others on cycle lengths in sparse graphs using the novel machinery of sublinear expanders.* Let us briefly describe some of these achievements.

A typical decomposition question asks whether the edges of some graph G can be partitioned into disjoint copies of another graph H — in which case we say that H *packs* into G. One of the oldest and best known conjectures in this area, posed by Ringel in 1963, concerns the decomposition of complete graphs into edge-disjoint copies of a fixed tree. More precisely, it states that every tree with n edges packs $2n+1$ times into the complete graph K_{2n+1}.

The earliest result about spanning tree decompositions is perhaps due to Walecki (1882), who showed that complete graphs on $2n$ vertices can be decomposed into spanning paths. After this, a number of increasingly sophisticated approaches, often using probabilistic techniques and the Regularity method, confirmed the decomposition for relaxations of the problem (such as considering specific classes of trees or approximate decompositions). Finally, in 2021, Alexey Pokrovskiy, Richard Montgomery and Benny Sudakov solved Ringel's conjecture for all sufficiently large cliques. Their argument involves, among many others, the absorption method (introduced by Rödl, Ruciński and Szemerédi [5]. This method plays an essential role in modern combinatorics, lying at the heart of breakthroughs such as Keevash's proof of the existence of designs.

Another major contribution of Montgomery is, incidently, the development of a particular type absorption technique, known as *distributive absorption* (see, e.g.,: [3, Section 5]). Absorption, roughly speaking, is a general name given to strategies for decomposing a large object O: firstly, one reserves a small substructure of O that has some flexibility in it; on a second step, the leftover structure is nearly completely decomposed; finally, the flexibility of the initially reserved structure is used to convert the approximate decomposition into a perfect one. Absorbers are often built using a combination of probabilistic and structural methods, and we shall not attempt to give details here (instead we refer the interested reader to the Montgomery's survey article [3, Section 5]).

Distributive absorption was developed by Montgomery in 2019, in an article confirming a conjecture of Kahn [2] about spanning trees of the binomial random graph $G(n,p)$. More precisely, he proved that for each $\Delta > 0$ there exists $C = C(\Delta)$ such that $G(n, C\log n/n)$ almost surely contains a copy of every tree with n vertices and maximum degree at most Δ.

We conclude this brief account by stating some groundbreaking progress obtained by Montgomery and collaborators on an old problem of Erdős. In 1975, Erdős asked what is the maximum number of edges that an n-vertex graph can have if it does not contain two edge-disjoint cycles on the same vertex set. Untill recently, the best known lower and upper bounds were, respectively, $\Omega(n \log\log n)$ and $\Omega(n^{3/2})$. Recently, Debsoumya Chakraborti, Oliver Janzer, Abhishek Methuku, and Richard Montgomery obtained an upper bound of order $n\cdot(n)$, closing the problem up to the polylog term. Their proof combines a number of novel methods — among which is a strategy for regularizing sparse graphs that might find applications in a number of related problems.

Altogether, the contributions mentioned above are but a small sample of many impressive contributions of Richard

Montgomery to Mathematics, certainly worthy of wide recognition!

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Canylo Radchenko receives the ECM Prize 2024, by Joaquim Ortega-Cerdà (Department of Mathematics and Computer Science, UB).

Danylo Radchenko has been awarded in the 9th ECM at Sevilla one of the ten EMS prizes for early career researchers for the construction of optimal spherical designs and his seminal input in the new field of Fourier interpolation, as well as for his fundamental contributions to the theory of polylogarithms.

Radchenko is a Ukrainian mathematician that studied his bachelor degree at the Taras Shevchenko National University of Kyv. He did a master degree at the University of Manitoba and in 2016 he completed his thesis under the supervision of D. Zaglier at the Max Plank Institute of Mathematics of Bonn. He did some postdoc stays at ETH in Zurich and at the International Center Abdus-Salam at Trieste. Since 2022 he holds a position of researcher at the CRM at the Laboratoire Paul Painlevé at Lille, in France.

According to his web page "My research interests include modular forms, algebraic number theory, convex and discrete geometry, combinatorics, as well as approximation theory and Fourier analysis."

This is a very wide range of interests! I will try to present some of his works. He was always showed a precocious interest in Mathematics. This started at a young age, and he has always been interested in Mathematical Competitions. He won a Gold medal in IMO in 2007 and while studying his bachelor he won the Grand First prize in the ICM for three consecutive years.

It is not surprising, then, that his first results were in graph theory and discrete geometry, that are topics that appear frequently in these competitions.

In 2016, he finished his thesis on functional equations of polylogarithms with a view to tackle the Zaglier conjecture that postulates that one can express the Dirichlet zeta function associated to a number field in terms of some determinants of polylogarithms of complex embeddings of the field.

But, even before that, he published two very influential papers in Annals of Mathematics in two completely different topics.

In the first one [4] in collaboration with A. Bondarenko and M. Viazoska, they solved a conjecture of Korevaar and Meyers. It is easy to state: They proved that for any degree N there is a collection of C_N points in the sphere of dimension d such that they form a spherical design for polynomials of degree N. A spherical design for the polynomials of degree at most N is a collection of C_N points X such that

$$\frac{1}{C_N}\sum_{x\in X} p(x) = \int_{\mathbb{S}^d} p(y)\, d\sigma(y)$$

for all polynomials p of degree N, where σ is the normalized measure on the sphere. The important point here is that one can pick $C_N \le CN^d$. The argument is through an ingenous fixed point theorem.

For this work and later works D. Radchenko received the Vasil Popov prize in approximation theory.

In the second one [3], a joint paper with H. Cohn, A. Kumar, S. Miller and M. Viazoska they proved the optimality of the Leech lattice among all 24-dimensional sphere packings. This was immediately after M. Viazovska had proved the analogous result in dimension 8.

All of this were side projects at the time of his graduate studies.

Later on he started his most ambitious program in my opinion. He found, together with M. Viazoska [2], a marvelous formula that allows to reconstruct a smooth decaying even function f knowing only a sequence of its values $\{f(\sqrt{n})\}_n$ and a sequence of the values of its Fourier transform $\hat{f}(\sqrt{n})$. It is a linear reconstructing formula written in terms of modular forms.

This was used to tackle a problem posed by Cohn et Kumar in 2006. They conjectured that the hexagonal lattice in dimension 2 is universally optimal. That is, it minimizes the potential energy for a large collection of potentials among all point configurations of a given density. In other words, the bee-cells are the most effective from the point of view of potential theory. Endowed with some variants of the Fourier interpolation formula, in a paper in the Journal of the AMS in 2022, [1], H. Cohn, A. Kumar, S. Miller, D. Radchenko and M. Viazovska solved the analogous problem in dimension eight and 24, that is they identified the optimal lattices. In dimension two the problem remains open, and it is the object of study of a recent ERC Starting Grant that was awarded to D. Radchenko.

For this later work and previous contributions to the theory of Fourier Interpolation, D. Radchenko has been recently awarded the Bronze medal of the CNRS.

On a personal note, I have had the opportunity to work with Danylo and if I had to highlight one of his mathematical traits is that he has the closest thing that I have seen to a Ramanujan-type intuition. He comes up with magical formulas that are possible to prove a posteriori but very difficult, for me at least, to even guess where they came from initially. At the same time, he is a very talented programmer, with a knack for modular forms, and he uses all the time numerical experiments to reinforce his intuition.

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Events

IMTech Colloquium 10/04/2024

by Gemma Huguet (DMAT, **IMTech**)

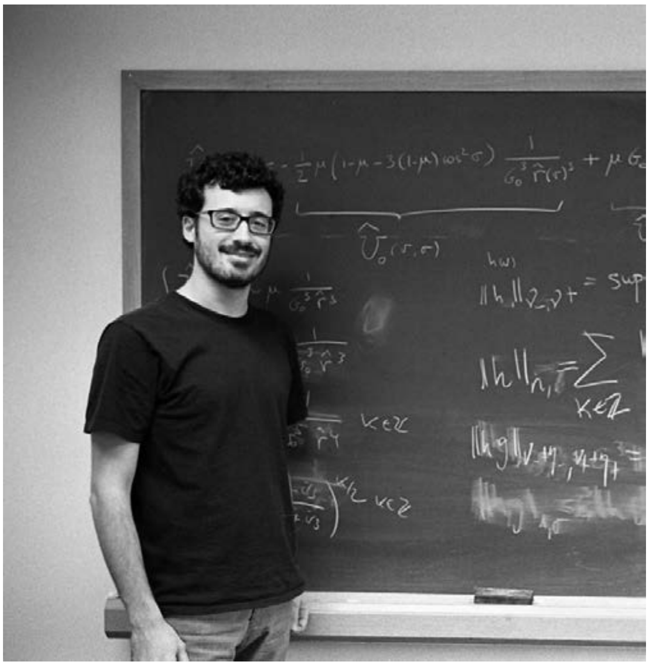
On April 10, 2024, Professor Marcel Guàrdia delivered the **IMTech**Colloquium Lecture at the Faculty of Mathematics and Statistics (FME). Professor Marcel Guàrdia is Full Professor at the Department of Mathematics and Computer Science at Universitat de Barcelona (UB) and affiliated at the Centre de Recerca Matemàtica (CRM). He was the Principal Investigator of the ERC Starting grant Haminstab (2018-2023) and he has been distinguished with an ICREA Academia Prize in 2018 and 2023. He is also the Scientific Director of the Maria de Maeztu award of the Centre de Recerca Matemàtica.

He obtained a PhD in Applied Mathematics from UPC in 2010. He has held post-doctoral positions at Penn State University, Fields Insitute (University of Toronto), University of Maryland at College Park, Institute for Advanced Study (Princeton) and Université de Paris 7 - Diderot. From 2017-2022, he was Associate Professor (Professor Agregat) at the UPC Mathematics Department (DMAT).

His research interests are Hamiltonian Systems, Celestial Mechanics, Hamiltonian Partial Differential Equations, Arnol'd diffusion, Exponentially small phenomena.

The title of his lecture was *Unstable motions in Celestial Mechanics*. He talked about one of the oldest problems in dynamical systems which is the stability of the Solar System. That is, consider N bodies moving following Newton's law of gravitation, one of them with large mass (the Sun) and the others with small masses (the planets). If one neglects, the gravitational interaction between planets, the classical Kepler's laws assert that the planets move on ellipses. Then, one wants to understand whether the effect of the planets mutual attraction causes long term changes on the shape and relative position of the Keplerian ellipses. Nowadays, it is known that the answer to the stability of the Solar system is rather nuanced and that stability and instability coexist for nearby initial conditions. In this talk, M. Guàrdia explained how to construct unstable motions in this model, which lead to drastic changes in the semimajor axes, eccentricity and inclination of these ellipses.

Results presented in the talk were highlighted in the article *New Proof Finds the 'Ultimate Instability' in a Solar System Model* that appeared in Quanta Magazine and the video recording of the talk is available at Zona video. ▷ Editorial



Interplays between algebra, combinatorics and proof formalization, by Marc Noy

On July 15th, 2024 took place at CRM the workshop Interplays between algebra, combinatorics and proof formalization. The goal was to cover the main ideas of the proof by Adiprasito, Huh and Katz [Hodge theory for combinatorial geometries, Annals Math. 2018] of the Rota-Hero-Welsh's Conjecture, proposed by Gian-Carlo Rota in the 1960s for graphs and later by Hero and Welsh for matroids, on the unimodality of the coefficients of the characteristic polynomial, or chromatic polynomial in the case of graphs.

There were four lectures in the morning by Anna de Mier on Matroids; Julian Pfeifle on Polytopes; Souvik Goswami on Cohomology; and Sebastià Xambó on Kähler packages and their combinatorial significance. In the afternoon there were three lectures on the proof assistant Lean: Algebra in Lean by María Inés de Frutos Fernández; Combinatorics in Lean by Yaël Dillies; and Geometry in Lean by Riccardo Brasca. It was orgganized by Marc Masdeu and Juanjo Rué in conjunction with CRM in Montreal. There were about 45 participants.

Ia Jornada IMTech 25/10/2024

by Gemma Huguet (DMAT, **IMTech**)

On October 25th, 2024, took place the Ia Jornada IMTech at the Faculty of Mathematics and Statistics (FME). The first IMTech meeting gathered around 80 participants, inclugind master and PhD students as well as several members of the mathematical community at UPC.

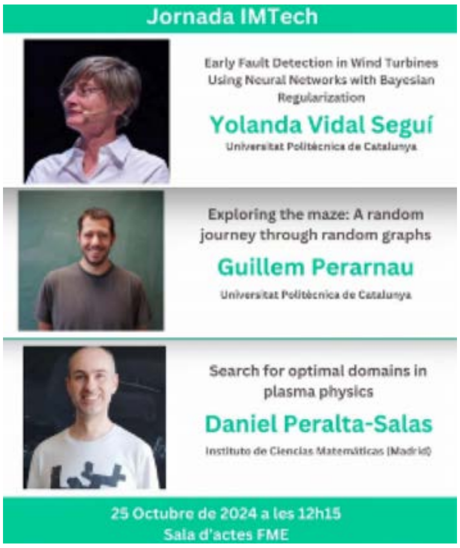
The morning featured two invited talks by members of **IMTech**. Yolanda Vidal Seguí (DMAT) kicked off the session with a presentation on *Early Fault Detection in Wind Turbines Using Neural Networks with Bayesian Regularization*, offering a compelling look at AI-driven predictive maintenance. Following this, Guillem Perarnau (DMAT) gave the talk *Exploring the Maze: A Random Journey Through Random Graphs*, where he discussed the challenges for theoretical analysis of random graph models.

A dynamic Poster Blitz session allowed presenters to introduce their work before the Poster Session, which showcased 12 research posters from master and PhD students at UPC covering a wide range of topics. Lively discussions continued over coffee, providing attendees with the opportunity to engage with presenters and deepen their understanding of the work on display.

The event continued with the Keynote Talk by Daniel Peralta-Salas (ICMAT), who is member of the scientific advisory board of **IMTech**. His talk was on *Search for Optimal Domains in Plasma Physics*.

The meeting concluded with a Prize Awards ceremony for the best posters, which recognized the contributions of Alexandra Lillo and Pau Blanco, PhD students at UPC, followed by a networking lunch at the FME garden, allowing participants to exchange ideas in a more informal setting.

The abstracts of the talks are available at Talks and the list of posters and presenters is available at Posters. The video recording of the talks is available at Zona video. ▷ Editorial



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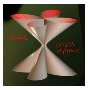

The *Eistüte* image in the cover,
from Herwig Hauser's Gallery of Singular Algebraic Surfaces,
is one of those included in the Imaginary exhibits.